A Software Reliability Model Using Fault Removal Efficiency

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Abstract

With the increase of human dependency over computer software, considerable effort has been given to determine software reliability effectively. A huge variety of software reliability growth models (SRGMs) have been developed to explain statistically how system reliability varies over time by monitoring the failure data sets during the testing process. The paper proposes a new SRGM based on taking into account the fault removal efficiency which is the ratio of corrected and detected faults during the testing process. The new model is compared to some known model from the relevant literature for two certain data sets and it turns out to perform better in terms of four GOF benchmarks.

Keywords: Reliability model, NHPP model, SRGM, software reliability engineering, project management.

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1 Introduction

The word — “Reliability” is somehow a qualitative term. In general, a product is reliable, if it delivers on its promises. Most engineering products are expected to function reliably once they have been developed, tested, and delivered. Software industry suffers many challenges in producing highly reliable software. A software development project cannot go off the rails; the project manager must adhere to a strict timeline and budget. On the one hand, management expects its testing team to eliminate all software flaws in order to increase software reliability. The management, on the other hand, wants to keep testing expenses to a minimum. These are two characteristics of software development that cannot be overlooked. As a result, management must make an informed decision about when the software will be released. Early release may result in less reliable software but lower testing expenses. A later release would result in more reliable software, but at the expense of increased testing expenditures. In such a scenario, the management prefers an optimal path or a trade-off option based on the considerations of reliability growth and testing cost. The primary objective of employing SRGMs is to test and debug the system until it achieves the necessary level of reliability. An SRGM is a statistical model that forecasts how software reliability will increase over time when faults are found and fixed [1, 2]. The strategy is based on the notion that a program has faults and these faults present themselves as visible failures during the testing process, which are identified and corrected. A crucial component of the development of these statistical models is recognizing realistic assumptions and then modelling the testing activities effectively within a specified or adequate analytical framework. Software reliability models offer a systematic and quantitative approach to figure out the failures in a timely manner. These models aid management in determining the amount of time and effort that should be spent on testing [3]. Since 1970s, a plenty of SRGMs have been developed under various sets of assumptions on testing environment [4]. The majority of the models have a common drawback that is unrealistic assumptions. They assume that the faults are independent and that the faults are corrected as soon as they are identified i.e. the number of detected faults is equal to the number of corrected faults [5]. These two assumptions do not hold true in a real-world testing scenario. The remedy of one fault may be contingent on the correction of another fault in the future. As a result, it is not always possible to remove all the identified faults immediately. Furthermore, it’s possible that when fixing the target fault, some other faults are rectified as well, which may have been
exposed in the future. It indirectly implies that detecting one fault leads to the correction of multiple faults. Thus, the number of repaired faults may be the same, higher, or lower than the number of detected faults in practise.

This paper presents a software reliability model that takes into account a special parameter “ω” called fault removal efficiency that maps between the number of identified faults and the number of repaired faults. The fault removal efficiency is defined as the fault removal rate per detected fault.

2 Related Work

The most commonly used technique in developing the SRGMs is Non-homogeneous Poisson process (NHPP) [5]. Other techniques include Bayesian models, Markov models etc. Many recent models [6–11] emphasize the use of different machine learning methods including neural network, fuzzy logic, deep learning etc. The Jelinski-Mornada model [12], regarded as the first SRGM was published in 1972. Since then, a lot of work has been done as the researchers shown a great interest in suggesting novel models that would best suit the failure data from the past. The literature on reliability modelling is large and extensive, as many researchers came up with a variety of models based on a variety of assumptions and techniques. Goel and Okumoto [13] developed a simple NHPP model that received a huge attention by the researchers. The Delayed S-Shaped model [14] is a variant of the NHPP process that divides the testing process into two different phases: fault detection phase and fault removal phase. The model incorporates a learning process as a result of the project team’s increasing experience and skill improvement. The Inflection S-Shaped model [15] is developed under the assumption that some faults are not exposed before some others are removed. The likelihood of detecting a failure at any given time is related to the number of detectable defects in the program at the time. Kapur and Garg [16] suggested a model based on the concept of dependent faults. While correcting the leading faults, several additional faults (called dependent faults) that may have caused future failure are also corrected. Kapur et al. [17] also presented two general frameworks for NHPP models: one for when the fault detection and fault elimination processes are supposed to be the same, and another for when they are supposed to be separate processes. Huang et al. [18] proposed a model for software reliability growth that integrates fault dependency and a time-dependent delay function. Peng et al. [19] designed a model that includes the fault detection and fault removal processes with the presence
of a testing effort function. Zhu and Pham [6] presented a software reliability model that took into account the fault-dependent detection and the imperfect fault removal. They used a genetic algorithm (GA) to estimate parameters in their model. Haque and Ahmad [20] proposed a similar type model that considers the issue of dependent faults in imperfect debugging environment. Haque and Ahmad [21] suggested another model that takes into account the uncertainty impact of the testing environment and assumptions when estimating software product reliability. Li and Pham [22] developed a model that uses fault introduction rate (i.e. number of new faults introduced per corrected fault) to represent the dependencies between fault detection and fault correction processes. The model additionally includes a testing coverage rate function. Chatterjee et al. [23] proposed a model that addresses the fault dependency issue in the context of multi-release software.

There are already a vast number of SRGMs, and it is impossible to include them all. We have tried to cover some models that address the issue of fault dependency. They are mainly based on imperfect debugging which provides the concept of fault introduction rate. These models simply consider $\varphi < 1$. There is one model known as Kapur-Garg model [16] that assumes $\varphi > 1$. However, most of the current SRGMs are based on the consideration that $\varphi = 1$. Our proposed model treats the fault detection and fault removal operations as two distinct processes and attempts to construct a link between them. The model is flexible to deal with any real value of $\varphi$ (i.e. $\varphi < 1$, $\varphi > 1$ and $\varphi = 1$).

3 Proposed Model

The proposed model is an NHPP model that is represented using a mean value function. Most of the SRGMs suggested till now are based on the assumptions that the faults are independent and they are fixed as soon as they are detected (i.e. the number of detected faults are equal to the number of corrected faults). They use a generalized function [17, 24]:

$$\frac{dm(t)}{dt} = b(t)[N - m(t)]$$

where,
- $m(t)$: estimated number of faults identified over time $t$.
- $N$: the total number of faults in the system before testing begins.
- $b(t)$: time dependent fault detection rate function.
The proposed model uses the concept of fault removal efficiency by incorporating a parameter \(\varphi\). It is the ratio of corrected and detected faults. Naturally, \(\varphi \geq 0\). Here, \(\varphi < 1\) means that the number of corrected faults is less than the number of detected faults and \(\varphi > 1\) means that the number of corrected faults is greater than the number of detected faults. \(\varphi = 1\) indicates that both are equal. Thus, the number of faults removed over time \(t\) is \(\varphi m(t)\) and Equation (1) can be framed as:

\[
\frac{dm(t)}{dt} = b(t)\left[N - \varphi m(t)\right]
\]

(2)

In this paper, we use the two-parameter logistic function to represent the fault detection rate. It is generally known as the inflection S-shaped fault detection rate function which has been used in many studies [14, 25]. It is expressed as follows:

\[
b(t) = \frac{b}{1 + \beta e^{-bt}}, \quad b > 0, \quad \beta > 0;
\]

(3)

where ‘\(b\)’ is a constant and ‘\(\beta\)’ is the inflection factor. Replacing the value of \(b(t)\) in Equation (2),

\[
\frac{dm(t)}{dt} = \frac{b}{1 + \beta e^{-bt}} \left[N - \varphi m(t)\right]
\]

(4)

Before testing begins, the number of detected faults and the number of corrected faults are both zero. Thus, \(\varphi = 1\). Solving Equation (4), for the mean value function \(m(t)\) with these initial conditions (i.e. \(m(0) = 0\) and \(\varphi = 1\)), we find:

\[
m(t) = \frac{N}{\varphi} \left[1 - \frac{1 + \beta}{\beta e^{bt} \varphi}\right]
\]

(5)

Table 1 presents the proposed model and some established models with their mean value functions.

### 4 Model Validation, Comparison and Analysis

The acceptability of a model is determined by identifying its strengths, weaknesses, and the level of trust that can be placed in the findings presented. SRGMs are evaluated using two major steps: first, parameter estimation and second, verification of the model fittings using different comparison criteria. The performance analysis of the new model is discussed in detail in the
Table 1  A set of reliability models

<table>
<thead>
<tr>
<th>Model</th>
<th>m(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G-O Model</td>
<td>$a(1-e^{-bt})$</td>
</tr>
<tr>
<td>DSS Model</td>
<td>$a(1-(1+bt)e^{bt})$</td>
</tr>
<tr>
<td>ISS Model</td>
<td>$a \left(1+\beta e^{-bt}\right)$</td>
</tr>
<tr>
<td>TC Model</td>
<td>$N\left(1-\left(\frac{\beta}{\beta+(at)^{\beta}}\right)^{\alpha}\right)$</td>
</tr>
<tr>
<td>Loglog Model</td>
<td>$N(1-e^{-(a^{b}-1)})$</td>
</tr>
<tr>
<td>New Model</td>
<td>$N \left[\frac{1 + \beta}{1 + \beta e^{bt}}\right]^{\varphi}$</td>
</tr>
</tbody>
</table>

following subsections. The model was compared to some existing models mentioned in Table 1.

4.1 Comparison Criteria

An SRGM is judged based on its potential to recreate the software’s actual behaviour and forecast the software’s future behaviour using past failure data. There are many Goodness-of-Fit (GOF) criteria available to compare the efficiency of different models and investigate how well a model fits a set of observations. In this paper, the model validation has been carried out using four GOF criteria namely – Mean square error or MSE, Predictive ratio risk or PRR, Coefficient of determination or $R^2$ and Akaike information criteria or AIC. The formulas of these criteria [21, 28] are given below where, ‘n’ denotes the number of samples in the dataset, and ‘p’ denotes the number of model parameters.

- **MSE**: The mean square error (MSE) represents the average squared residual (i.e. the difference between the observed value and predicted value), and is defined as:

  $$\text{MSE} = \frac{\sum_{i=1}^{n} (m_i - m(t_i))^2}{n - p}$$
• **PRR**: The predictive-ratio risk (PRR) calculates the error per model-estimate and it is defined as:

\[
PRR = \sum_{i=1}^{n} \left( \frac{m(t_i) - m_i}{m(t_i)} \right)^2
\]

• **\(R^2\)**: It measures the distribution of data points around the fitted regression line. This evaluation criteria is also known as the coefficient of determination:

\[
R^2 = 1 - \frac{\sum_{i=1}^{n} (m_i - m(t_i))^2}{\sum_{i=1}^{n} (m_i - \frac{\sum_{j=1}^{n} m_j}{n})^2}
\]

• **AIC**: AIC is used to determine which of several models is most likely to be the best for a given dataset by providing a score value that penalizes the number of model parameters. The well known formula of AIC is:

\[
AIC = 2p - 2 \log(L)
\]

where, \(L\) represents the maximum likelihood estimate. An alternate measure for LSE regression is as follows [29]:

\[
AIC = n \times \ln(MSE) + 2p
\]

The smaller MSE, PRR, and AIC values, as well as the greater \(R^2\) value that tends to 1, are always anticipated to justify fewer fitting flaws and improved model performance [27].

**4.2 Dataset Used**

In [30], the failure datasets of four different releases of a Tandem Computer Project have been reported. We used the failure data of first two releases for our model validation and comparison. In release-1, after 20 weeks of testing and 10000 hrs of CPU execution, total 100 errors were collected. In release-2, total 120 errors were reported over the testing duration of 19 weeks and 10272 CPU Hrs. The datasets of Release-1 and Release-2 are shown together in Table 2.

**4.3 Results and Comparison**

The fittings of software reliability models are determined only when it is feasible to estimate their parameters. Parameter estimation refers to the
Table 2  Tandem computers test data

<table>
<thead>
<tr>
<th>Test Week</th>
<th>Cumulative Faults</th>
<th>Cumulative Faults</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Release-1</td>
<td>Release-2</td>
</tr>
<tr>
<td>1</td>
<td>16</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>24</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>27</td>
<td>26</td>
</tr>
<tr>
<td>4</td>
<td>33</td>
<td>34</td>
</tr>
<tr>
<td>5</td>
<td>41</td>
<td>40</td>
</tr>
<tr>
<td>6</td>
<td>49</td>
<td>48</td>
</tr>
<tr>
<td>7</td>
<td>54</td>
<td>61</td>
</tr>
<tr>
<td>8</td>
<td>58</td>
<td>75</td>
</tr>
<tr>
<td>9</td>
<td>69</td>
<td>84</td>
</tr>
<tr>
<td>10</td>
<td>75</td>
<td>89</td>
</tr>
</tbody>
</table>

process of making efficient use of sample data in estimating the values of unknown variables that exist in the mathematical models [31]. There are several techniques available to estimate the parameters, for example, least square estimation, maximum likelihood estimation, Bayes parameter estimation etc. Generally, LSE is recommended when the sample size is small and censoring is not particularly heavy [32, 33]. Thus, we have preferred Least Square Estimation (LSE) approach to decide the parameters’ values of all six models (listed in Table 1).

Then we can calculate the aforementioned four GOF criteria of all SRGMs using the parameter values. For the Tandem project release-1 dataset, Table 3 provides the results including the estimated parameter values and criteria values. Similarly, Table 4 presents the results for release-2 dataset. The following is a summary of the Table 3:

- **MSE = 1.936.** It is nearly four times lower than the second best value achieved (i.e. Loglog model – 8.437) in this study.
- **PRR= 0.026.** It is roughly 8-times lower than the second best value for G-O model.
- **R² = 0.998.** It is the highest among all models.
- **AIC = 10.751 which is the smallest among all models.**

Table 4 can be summarized as:

- **MSE = 3.77.** It is near about half of the second best value achieved (i.e. ISS model – 7.139) in this study.
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Table 3  Model validation on release-1 dataset

<table>
<thead>
<tr>
<th>Model</th>
<th>LSEs</th>
<th>MSE</th>
<th>PRR</th>
<th>$R^2$</th>
<th>AIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>G-O Model</td>
<td>a = 130.2, b = 0.083</td>
<td>12.915</td>
<td>0.203</td>
<td>0.986</td>
<td>51.061</td>
</tr>
<tr>
<td>DSS Model</td>
<td>a = 103.984, b = 0.265</td>
<td>28.066</td>
<td>1.084</td>
<td>0.969</td>
<td>66.584</td>
</tr>
<tr>
<td>ISS Model</td>
<td>a = 110.829, b = 0.172, $\beta$ = 1.205</td>
<td>10.564</td>
<td>0.305</td>
<td>0.989</td>
<td>47.899</td>
</tr>
<tr>
<td>TC Model</td>
<td>N = 119.205, a = 1.3798 × 10^{-3}, b = 1.111, $\beta$ = 7.337, $\alpha$ = 65.069</td>
<td>14.577</td>
<td>0.3</td>
<td>0.987</td>
<td>55.836</td>
</tr>
<tr>
<td>Loglog Model</td>
<td>N = 105.109, a = 1.095, b = 0.947</td>
<td>8.437</td>
<td>0.238</td>
<td>0.991</td>
<td>43.403</td>
</tr>
<tr>
<td>New Model</td>
<td>N = 110.382, b = 0.287, $\beta$ = 7.624, $\varphi$ = 1.069</td>
<td>1.936</td>
<td>0.026</td>
<td>0.998</td>
<td>10.751</td>
</tr>
</tbody>
</table>

Table 4  Model validation on release-2 dataset

<table>
<thead>
<tr>
<th>Model</th>
<th>LSEs</th>
<th>MSE</th>
<th>PRR</th>
<th>$R^2$</th>
<th>AIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>G-O Model</td>
<td>a = 182.95, b = 0.061</td>
<td>26.217</td>
<td>0.212</td>
<td>0.982</td>
<td>59.948</td>
</tr>
<tr>
<td>DSS Model</td>
<td>a = 127.399, b = 0.242</td>
<td>14.693</td>
<td>0.796</td>
<td>0.990</td>
<td>48.946</td>
</tr>
<tr>
<td>ISS Model</td>
<td>a = 124.445, b = 0.254, $\beta$ = 3.778</td>
<td>7.139</td>
<td>0.261</td>
<td>0.996</td>
<td>35.08</td>
</tr>
<tr>
<td>TC Model</td>
<td>N = 128.17, a = 0.02, b = 1.442, $\beta$ = 7.05, $\alpha$ = 83.734.</td>
<td>12.913</td>
<td>0.45</td>
<td>0.993</td>
<td>45.805</td>
</tr>
<tr>
<td>Loglog Model</td>
<td>N = 119.881, a = 1.06, b = 1.157</td>
<td>7.956</td>
<td>0.263</td>
<td>0.995</td>
<td>37.139</td>
</tr>
<tr>
<td>New Model</td>
<td>N = 124.553, b = 0.314, $\beta$ = 8.258, $\varphi$ = 1.032</td>
<td>3.77</td>
<td>0.016</td>
<td>0.998</td>
<td>22.724</td>
</tr>
</tbody>
</table>

- PRR = 0.016. It is nearly 13-times lower than the second best value achieved (i.e. 0.212) in this study.
- $R^2$ = 0.998 again. It is the highest among all models.
- AIC = 22.724 which is the smallest among all models.

Thus on both the datasets, the proposed model achieved the best fitting values with the lowest MSE, PRR, AIC and the highest $R^2$. Figures 1 and 2 display the curves representing the model performance on each dataset, comparing the estimated faults to the observed faults.
Then we can calculate the aforementioned four GOF criteria of all SRGMs using the parameter values. For the Tandem project release-1 dataset, Table 4.2 provides the results including the estimated parameter values and criteria values. Similarly, Table 4.3 presents the results for release-2 dataset.

The following is a summary of the Table 4.2:

- **MSE = 1.936.** It is nearly four times lower than the second best value achieved (i.e. Loglog model - 8.437) in this study.
- **PRR = 0.026.** It is roughly 8-times lower than the second best value for G-O model.
- **R² = 0.998.** It is the highest among all models.
- **AIC = 10.751 which is the smallest among all models.**

Table 4.3 can be summarized as:

- **MSE = 3.77.** It is near about half of the second best value achieved (i.e. ISS model – 7.139) in this study.
- **PRR = 0.016.** It is nearly 13-times lower than the second best value achieved (i.e. 0.212) in this study.
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- **AIC = 22.724 which is the smallest among all models.**

Thus on both the datasets, the proposed model achieved the best fitting values with the lowest MSE, PRR, AIC and the highest R².

Figure 4.1 and Figure 4.2 display the curves representing the model performance on each dataset, comparing the estimated faults to the observed faults.

5. CONCLUSION

The new model suggested in the paper, establishes a relationship between fault detection and fault correction processes assuming that they are two different processes. It incorporates the widely used inflection S-shaped fault detection rate function and a parameter that represents the fault removal efficiency. The model has been validated on two real world datasets and...
compared its Goodness-of-fit with five reputed models using four evaluation criteria. In all four benchmarks, the findings clearly indicate that the proposed model beats the other five models. The accuracy of SRGMs heavily depends on the types of datasets. A model may perform well on some datasets while failing to deliver good results on others. Therefore, it will be premature to claim the superiority of the model. In the future, we will concentrate on a thorough analysis of the model using a wide range of datasets and other comparison criteria.

References


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Biographies

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