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CLASSICAL STATIC SYSTEM RELIABILITY AND ADJUSTED STATIC SYSTEM RELIABILITY WITH PRIOR & POSTERIOR VARIATIONS

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Abstract

This paper considers an important concept which suggested to take stock of the over estimation in reliability characteristics or under estimation of hazard rate. Using this concept, the study considers the analysis of the reliability characteristics of an exponential lifetime model when prior variations in its parameters are suspected.

Key Words: Robustness**,** adjustment factor, updated and predictive basic distributions

1. Introduction

In life testing the experiments are conducted to record the lifetime data and the same are then used for analyzing the systems in terms of their reliability characteristics like-reliability or survival function, hazard rate, MTSF etc. in the classical setup. Some of the studies dealing with this aspect are Mann, Schafer & Singhpurwala [7], Kapur and Lemberson [4], Lawless [5], Sinha [12] etc. However, the life testing experiments are costly and time consuming phenomenon, and, therefore, it should be recognized that the parameters, characterizing reliability characteristics, in a life time distribution, are bound to follow some random variations due to continuous environmental stresses on the system. So it is a factor which should be considered with experimental data for analyzing the system reliability characteristics.

Obviously, therefore, the reliability characteristics need to be analyzed in the Bayesian setup. Some of the comprehensive studies dealing with this aspect of the problem are Savage [9], Lindley [6], Bhattacharya [1], Box and Tiao [2], and Martz & Wallher [8]. Studies like Sharma et. al. [10, 11] are also efforts in the same direction.

In the Bayesian analysis of reliability characteristics, the parameter(s), representing reliability characteristics, in a lifetime distribution follows random variations represented by priors. The priors then are updated with experimental data. The updated form of the prior is then called the posterior distribution which becomes the basis of future analysis of the problem involved.

However, in same study G.G. Brush [3] showed that even the lifetime distributions can be updated in view of variations in its parameters. The updated basic distribution has been used to study the robustness of the reliability characteristics when parametric variations in the parameters of the lifetime distributions are suspected.

In the process of analysis, we still come through yet another problem of interest. Here if should be recognized that the over estimation of system reliability or under estimation of hazard rate is a bad phenomenon. After using the updated and predictive basic distributions, we get three important distributions whose comparison leads to the analysis of robust character of lifetime distributions when variations in the parameters are suspected. In this regard, an adjustment factors has been suggested to take stock of the over estimation in good reliability characteristics like system reliability or under estimation of some bad reliability characteristics like the hazard rate. A methodology has been suggested which helps in adjusting the over or under estimation phenomenon. The discussions of the problem also highlight an important concept where classical reliability is adjustable with Bayesian information. It has been shown as to how inverse reliability can be used as an adjustment factor so that an over estimate of classical reliability or under estimation of hazard rate can be associated with the Bayesian estimation of survival function or hazard rate.

In the light of above discussions, the present paper deals with the development of a methodology of developing an adjustment factor for connecting the reliability characteristics by using the Bayesian information available. The theoretical developments have been highlighted with suitable example.

2. Notations, Assumptions and Definitions

(a)

$f(¥) = pdf$	Probability density function
$P[X > t \mid \theta \leq \theta_0]$	Classical System Reliability (CSR)
$P[\theta \leq \theta_0 \mid X > t]$	Adjustment factor for system reliability (AFSR)
CSR^*, $AFSR^*$	Respective reliability of Classical and Adjustment factor for system reliability under Prior information.
CSR^{**}, $AFSR^{**}$	Respective reliability of Classical and Adjustment factor for system reliability under Posterior information.
$r.v.$	Random variable
CV	Coefficient of Variation
$MTSF$	Mean time to system failure
$\theta \leq \theta_0$	Span for MTSF
$R(t) = P[X \geq t]$	Reliability of the independent and identical components for a testing time t
$R_{km}(t), R_p(t)$ & $R_s(t)$	Respective reliability of k-out-of-m, parallel and series system for the mission time t.

(b) It is assumed that the components are identical and statistically independent.

(c) k-out-of-m system: This system consists of m-components and operates as a long as any of its k ($\leq$m) components operate.
With simple probabilistic reasoning,

$$R_{km}(t) = \sum_{\mu=\kappa}^{m} \binom{m}{\mu} [R(t)]^{\mu} \; [1-R(t)]^{(m-\mu)} \tag{1}$$

For $k = 1$, the system reduces to a parallel system which functions as long as any of its components operates. Thus,

$$R_p(t) = 1-[1-R(t)]^m \tag{2}$$

For $k = m$, the system reduces to series system which fails as soon as any of its components fails.
As such (1) reduces to

$$R_s(t) = [R(t)]^m \tag{3}$$

3. Statistical Backgrounds

It is assumed that :
(a) In the classical set up, θ is assumed as constant, however it is taken as r.v. in the Bayesian setup. Thus, θ perform as a random variable.
(b) The basic lifetime distribution for each component is exponential, $\exp(1/\theta)$ with p.d.f.

$$f(x,\theta) = \frac{1}{\theta}\exp(-x/\theta) \qquad ; x,\ \theta > 0 \tag{4}$$

Thus for a component,

$$\text{MTSF} = E[X] = \theta\ , \qquad V(X) = \theta^2$$

$$R(t) = P[X \geq t] = \exp(-t/\theta)$$

(c) 'θ' is a random variable and its prior is gamma distribution with p.d.f.

$$g(\theta) = \frac{\theta^{\lambda-1}\ \exp(-\theta)}{\lceil\lambda} \quad ; \qquad \lambda > 0, \quad 0 < \theta < \infty \tag{5}$$

(d) Let, $X = (X_1, X_2, \ldots\ldots, X_n)$ be a random sample from $exp(1/\theta)$. Then the posterior distribution of θ, in respect of its prior in (5), will be

$$\Pi(\theta\,|\,\underset{\sim}{x}) = \frac{\theta^{\lambda-n-1}\exp\left(-\sum_{i=1}^{n} x_i/\theta\right)\exp(-\theta)}{\int_0^\infty \theta^{\lambda-n-1}\exp\left(-\sum_{i=1}^{n} x_i/\theta\right)\exp(-\theta)\,d\theta} \tag{6}$$

(e) If $(X_1, X_2, \ldots\ldots\ldots, X_n)$ are identically distributed, as exponential with parameter $(1/\theta)$, then $Y = Min(X_1, X_2, \ldots\ldots\ldots, X_n)$ is also exponentially distributed with parameter $(1/\theta)$, with p.d.f.

$$f(y) = \frac{n}{\theta}\exp(-yn/\theta) \qquad ; y, \theta > 0 \tag{7}$$

Thus,

$$R_s(t) = \exp(-nt/\theta)$$

(f) If $X_1, X_2, \ldots\ldots\ldots, X_n$ are identically distributed exponential with parameter $(1/\theta)$, then $Z = Max(X_1, X_2, \ldots\ldots\ldots, X_n)$ has the p.d.f.

$$f(z) = \frac{n}{\theta}\exp(-z/\theta)(1 - \exp(-z/\theta))^{n-1} \qquad ; z, \theta > 0 \tag{8}$$

Thus,

$$R_p(t) = 1 - (1 - \exp(-t/\theta))^n$$

4. Comparisons between Classical Reliability and an Adjustment Factor under Prior Information

For, the purpose of making comparison between the classical and adjusted reliability characteristics, an additional concept is introduced.

4.1 Classical Analysis of Reliability Function for Various Static System Models

In the classical setup the system reliability can be defined as-

$$P[X > t | \theta \le \theta_0] = \frac{P[X > t \text{ and } \theta \le \theta_0]}{P[\theta \le \theta_0]} \tag{9}$$

Equation (9) give us the proportion of the systems which survive for the mission time t when 'θ' is a random variable defined in the span for MTSF$\theta \le \theta_0$.

(i) On using equations (4) and (5), the classical reliability function $R(t)$, say, $CSR^*(t)$, is

$$CSR^*(t) = \frac{\int_0^{\theta_0} R(t) g(\theta) d\theta}{\int_0^{\theta_0} g(\theta) d\theta}$$

$$CSR^*(t) = \frac{\int_0^{\theta_0} \theta^{\lambda-1} \exp(-t/\theta) \exp(-\theta) d\theta}{\int_0^{\theta_0} \theta^{\lambda-1} \exp(-\theta) d\theta} \tag{10}$$

(ii) Similarly, on using equations (5) and (7), the classical reliability $R_s(t)$, say, $CSR_s^*(t)$, becomes

$$CSR_s^*(t) = \frac{\int_0^{\theta_0} R_s(t) g(\theta) d\theta}{\int_0^{\theta_0} g(\theta) d\theta}$$

$$CSR_s^*(t) = \frac{\int_0^{\theta_0} \theta^{\lambda-1} \exp(-nt/\theta) \exp(-\theta) d\theta}{\int_0^{\theta_0} \theta^{\lambda-1} \exp(-\theta) d\theta} \tag{11}$$

(iii) Similarly, on using equations (5) and (8), the classical reliability $R_p(t)$, say, $CSR_p^*(t)$, will be

$$CSR_p^*(t) = \frac{\int_0^{\theta_0} R_p(t) g(\theta) d\theta}{\int_0^{\theta_0} g(\theta) d\theta}$$

$$CSR_p^*(t) = \frac{\int_0^{\theta_0} \theta^{\lambda-1} \exp(-\theta)[1-(1-\exp(-t/\theta))^n] d\theta}{\int_0^{\theta_0} \theta^{\lambda-1} \exp(-\theta) d\theta} \tag{12}$$

4.2 Adjustment Factor for the Various Static Models

The adjustment factor may we written as-

$$P[\theta \le \theta_0 \mid X > t] = \frac{P[X > t \mid \theta \le \theta_0] P[\theta \le \theta_0]}{P[X > t]}$$

$$= \frac{P[X > t \; and \; \theta \le \theta_0]}{P[X > t]} \tag{13}$$

The Bayesian probability or Inverse probability defined in this case can be considered as the adjustment factor for the classical reliability. It gives answer to the question that:-

(a) If the MTSF varies in the span $\theta \le \theta_0$, what percentage of systems will survive the mission time t.

(b) How much be proportion of the system will be surviving the mission time t?

(i) On using equations (4) and (5), adjustment factor for system reliability function $R(t)$, say, $AFSR^*(t)$, will be

$$AFSR^*(t) = \frac{\int_0^{\theta_0} R(t)\, g(\theta)\, d\theta}{\int_0^{\infty} R(t) g(\theta) d\theta}$$

$$AFSR^*(t) = \frac{\int_0^{\theta_0} \theta^{\lambda-1} e^{(-t/\theta)} e^{(-\theta)} d\theta}{\int_0^{\infty} \theta^{\lambda-1} e^{(-t/\theta)} e^{(-\theta)} d\theta} \tag{14}$$

(ii) Similarly, on using equations (5) and (7), adjustment factor for system reliability of $R_s(t)$, say, $AFSR_s^*(t)$, becomes

$$AFSR_s^*(t) = \frac{\int_0^{\theta_0} R_s(t) g(\theta) d\theta}{\int_0^{\infty} R_s(t) g(\theta) d\theta}$$

$$AFSR_s^*(t) = \frac{\int_0^{\theta_0} \theta^{\lambda-1} e^{(-nt/\theta)} e^{(-\theta)} d\theta}{\int_0^{\infty} \theta^{\lambda-1} e^{(-nt/\theta)} e^{(-\theta)} d\theta} \tag{15}$$

(iii) On using equations (5) and (8), the adjustment factor for system reliability of $R_p(t)$, say, $AFSR_p^*(t)$, is

$$AFSR_p^*(t) = \frac{\int_0^{\theta_0} R_p(t) g(\theta) d\theta}{\int_0^{\infty} R_p(t) g(\theta) d\theta}$$

$$AFSR_p^*(t) = \frac{\int_0^{\theta_0} \theta^{\lambda-1} e^{(-\theta)} [1-(1-e^{-t/\theta})^n] d\theta}{\int_0^{\infty} \theta^{\lambda-1} e^{(-\theta)} [1-(1-e^{-t/\theta})^n] d\theta} \tag{16}$$

5 Comparisons between Classical Reliability and an Adjustment Factor for System Reliability under Posterior Information

5.1 Classical Analysis of Reliability Functions for Various Static Models

In the classical approach this may be written as-

$$P[X > t \mid \theta \le \theta_0] = \frac{P[X > t \text{ and } \theta \le \theta_0]}{P[\theta \le \theta_0]} \tag{17}$$

(i) Using equations (4) and (6), the classical analysis of the reliability function $R(t)$, say, $CSR^{**}(t)$, can be attained from the expressions

$$CSR^{**}(t) = \frac{\int_0^{\theta_0} R(t)\Pi(\theta \mid \underset{B}{x})d\theta}{\int_0^{\theta_0} \Pi(\theta \mid \underset{B}{x})d\theta}$$

$$CSR^{**}(t) = \frac{\int_0^{\theta_0} \theta^{\lambda-n-1} \exp\left(-\sum_{i=1}^{n} x_i + t/\theta\right) \exp(-\theta)d\theta}{\int_0^{\theta_0} \theta^{\lambda-n-1} \exp\left(-\sum_{i=1}^{n} x_i/\theta\right) \exp(-\theta)d\theta} \tag{18}$$

(ii) Similarly, on using equations (6) and (7), the classical analysis of $R_s(t)$, say, $CSR_s^{**}(t)$, can be performed from

$$CSR_s^{**}(t) = \frac{\int_0^{\theta_0} R_s(t)\Pi(\theta \mid \underset{B}{x})d\theta}{\int_0^{\theta_0} \Pi(\theta \mid \underset{B}{x})d\theta}$$

$$CSR_s^{**}(t) = \frac{\int_0^{\theta_0} e^{-(nt/\theta)} \theta^{\lambda-n-1} \exp\left(-\sum_{i=1}^{n} x_i/\theta\right) \exp(-\theta)d\theta}{\int_0^{\theta_0} \theta^{\lambda-n-1} \exp\left(-\sum_{i=1}^{n} x_i/\theta\right) \exp(-\theta)d\theta} \tag{19}$$

(iii) Similarly, on using equations (6) and (8), the classical analysis of $R_p(t)$, say, $CSR_p^{**}(t)$, can be achieved from

$$CSR_p^{**}(t) = \frac{\int_0^{\theta_0} R_p(t)\Pi(\theta \mid \underset{B}{x})d\theta}{\int_0^{\theta_0} \Pi(\theta \mid \underset{B}{x})d\theta}$$

$$CSR_p^{**}(t) = \frac{\int_0^{\theta_0} [1-(1-e^{-t/\theta})^n]\theta^{\lambda-n-1} \exp\left(-\sum_{i=1}^{n} x_i/\theta\right) \exp(-\theta)d\theta}{\int_0^{\theta_0} \theta^{\lambda-n-1} \exp\left(-\sum_{i=1}^{n} x_i/\theta\right) \exp(-\theta)d\theta} \tag{20}$$

5.2 Adjusted Analysis of Reliability Functions for Various Static System Models

We know that

$$P[\theta \le \theta_0 \,|\, X > t] = \frac{P[X > t \,|\, \theta \le \theta_0] P[\theta \le \theta_0]}{P[X > t]} = \frac{P[X > t \; and \; \theta \le \theta_0]}{P[X > t]}$$

(i) On using equations (4) and (6), the adjustment factor for system reliability $R(t)$, say, $AFSR^{**}(t)$, can be performed from

$$AFSR^{**}(t) = \frac{\int_0^{\theta_0} R(t)\Pi(\theta \,|\, \underset{\sim}{x})d\theta}{\int_0^{\infty} R(t)\Pi(\theta \,|\, \underset{\sim}{x})d\theta} \tag{21}$$

$$AFSR^{**}(t) = \frac{\int_0^{\theta_0} \theta^{\lambda-n-1} \exp\left(-\sum_{i=1}^{n} x_i + t/\theta\right) \exp(-\theta)d\theta}{\int_0^{\infty} \theta^{\lambda-n-1} \exp\left(-\sum_{i=1}^{n} x_i + t/\theta\right) \exp(-\theta)d\theta} \tag{22}$$

(ii) Similarly, on using equations (5) and (6), the adjustment factor for system reliability $R_s(t)$, say, $AFSR_s^{**}(t)$, can be performed from

$$AFSR_s^{**}(t) = \frac{\int_0^{\theta_0} R_s(t)\Pi(\theta \,|\, \underset{\sim}{x})d\theta}{\int_0^{\infty} R_s(t)\Pi(\theta \,|\, \underset{\sim}{x})d\theta}$$

$$AFSR_s^{**}(t) = \frac{\int_0^{\theta_0} \theta^{\lambda-n-1} \exp\left(-\sum_{i=1}^{n} x_i + nt/\theta\right) \exp(-\theta)d\theta}{\int_0^{\infty} \theta^{\lambda-n-1} \exp\left(-\sum_{i=1}^{n} x_i + nt/\theta\right) \exp(-\theta)d\theta} \tag{23}$$

(iii) On using equations (6) and (8), the adjustment factor for system reliability $R_p(t)$, say, $AFSR_p^{**}(t)$, can be done by using

$$AFSR_p^{**}(t) = \frac{\int_0^{\theta_0} R_p(t)\Pi(\theta \,|\, \underset{\sim}{x})d\theta}{\int_0^{\infty} R_p(t)\Pi(\theta \,|\, \underset{\sim}{x})d\theta}$$

$$AFSR_p^{**}(t) = \frac{\int_0^{\theta_0} [1-(1-e^{-t/\theta})^n]\theta^{\lambda-n-1} \exp\left(-\sum_{i=1}^{n} x_i/\theta\right) \exp(-\theta)d\theta}{\int_0^{\infty} [1-(1-e^{-t/\theta})^n]\theta^{\lambda-n-1} \exp\left(-\sum_{i=1}^{n} x_i/\theta\right) \exp(-\theta)d\theta} \tag{24}$$

6. Illustrations

6.1 Comparison between Classical and Adjusted System Reliability When an Adjusted Factor under Prior Information is used

For demonstrating variations in classical and adjusted system reliability as the mean of quality distribution and the mission time varies, we use the expressions in equations (10) and (14). The respective values of $CSR^{*}(t)$ and $AFSR^{*}(t)$ are given in Table 1.

Now for analyzing the behavior of classical system reliability and adjusted system reliability, when the components are arranged in the form of series and parallel configuration. We consider n=10 (number of components) and use the expressions in equations [(11) (12), (15) and (16)], the respective values of $CSR_{s}^{*}(t)$, $AFSR_{s}^{*}(t)$ and $CSR_{p}^{*}(t)$, $AFSR_{p}^{*}(t)$ are summarized in Tables (2-3).

6.2 Comparison between Classical, Adjusted System Reliability and Adjusted Factor under Posterior Information

Estimates following the methodology in the previous section, the reliability functions for various static system models with posterior information. These processes have been listed in Table 4, 5 and 6. A sample of size 10 was generated from the p.d.f. in (4) with parameters (quality mean θ =1, 2), and then getting the sum of all the observations as $(\sum_{i=1}^{n} x_i = 9.64, \ \sum_{i=1}^{n} x_i = 21.33)$ respectively to get the reliability estimates in both the cases. The number of components is also 10. Using expressions in equations [(18, 22), (19, 23) and (20, 24)], the respective values of [$\{CSR^{**}(t), AFSR^{**}(t)\}$, $\{CSR_{s}^{**}(t), AFSR_{s}^{**}(t)\}$ and $\{CSR_{p}^{**}(t), AFSR_{p}^{**}(t)\}$] are summarized in Tables [4, 5 and 6].

7. Conclusions

After highlighting some of the concepts regarding robustness of reliability function, when parameters in the lifetime distribution are considered as random variable in the Bayesian setup, yet another concept of over and under estimation of reliability and hazard rate have been proposed. A methodology has been developed to deal with the phenomenon by using an adjustment factor based on Bayesian information. Subsequently, it is shown as to how we can use the Bayesian information to adjust the classical knowledge; various tables can be analyzed to see the robust character of classical estimates as adjusted in the light of Bayesian knowledge.

From Table- (1, 2,3,4,5 and 6), it is seen that

(i) As usual the, reliability for different static system models decreases uniformly as the mission time t increases.

(ii) CSR^{*} And $AFSR^{*}$ uniformly increases as the span of MTSF, $\theta \le \theta_0$ increases. Accordingly, the resulting adjustment in classical reliability tends to be uniformly less.

(iii) Resulting tends in classical reliability can also be ascertained with the resulting adjustment factor.

Other trends can be analyzed in all the situations by going through the Tables (1-6).

Time	$\theta_0 = 1,$	$\lambda = 1$		$\theta_0 = 2,$	$\lambda = 1$	
t	CSR^*	$AFSR^*$	$CSR^* \times AFSR^*$	CSR^*	$AFSR^*$	$CSR^* \times AFSR^*$
0.01	0.93	0.62	0.57	0.95	0.86	0.81
0.05	0.78	0.58	0.45	0.83	0.84	0.70
0.10	0.66	0.55	0.36	0.74	0.83	0.61
0.15	0.58	0.52	0.29	0.66	0.82	0.54
0.20	0.51	0.49	0.25	0.60	0.81	0.49
0.25	0.45	0.47	0.21	0.55	0.79	0.44
0.30	0.40	0.45	0.18	0.51	0.78	0.40
0.35	0.36	0.43	0.16	0.47	0.77	0.37
0.40	0.33	0.41	0.13	0.44	0.76	0.34
0.45	0.29	0.40	0.12	0.41	0.75	0.31
0.50	0.27	0.38	0.10	0.38	0.74	0.29

Table 1: Comparison between Classical Reliability and Adjustment Factor for System Reliability $(R(t))$ under Prior Information

Time	$\theta_0 = 1,$ $\lambda = 1,$			$\theta_0 = 2,$	$\lambda = 1$	
t	CSR_s^*	$AFSR_s^*$	$CSR_s^* \times AFSR_s^*$	CSR_s^*	$AFSR_s^*$	$CSR_s^* \times AFSR_s^*$
0.01	0.66	0.55	0.36	0.74	0.83	0.61
0.05	0.27	0.38	0.10	0.38	0.75	0.29
0.10	0.11	0.26	0.03	0.21	0.66	0.14
0.15	0.05	0.18	0.01	0.13	0.59	0.08
0.20	0.03	0.12	0.00	0.08	0.52	0.04
0.25	0.01	0.08	0.00	0.06	0.46	0.03
0.30	0.00	0.06	0.00	0.04	0.41	0.02
0.35	0.00	0.04	0.00	0.03	0.36	0.01
0.40	0.00	0.03	0.00	0.02	0.32	0.00
0.45	0.00	0.02	0.00	0.01	0.28	0.00
0.50	0.00	0.01	0.00	0.00	0.24	0.00

Table 2: Comparison between Classical Reliability and Adjustment Factor for System Reliability $(R_s(t))$ under Prior Information

Time	$\theta_0 = 1,$	$\lambda = 1$		$\theta_0 = 2,$	$\lambda = 1$	
T	CSR_p^*	$AFSR_p^*$	$CSR_p^* \times AFSR_p^*$	CSR_p^*	$AFSR_p^*$	$CSR_p^* \times AFSR_p^*$
0.01	0.99	0.63	0.62	0.99	0.86	0.86
0.05	0.97	0.62	0.60	0.97	0.86	0.84
0.10	0.94	0.61	0.57	0.95	0.85	0.81
0.15	0.91	0.60	0.55	0.93	0.85	0.78
0.20	0.88	0.60	0.52	0.91	0.85	0.77
0.25	0.85	0.59	0.50	0.88	0.85	0.75
0.30	0.84	0.58	0.48	0.86	0.84	0.73
0.35	0.79	0.57	0.45	0.84	0.84	0.71
0.40	0.77	0.56	0.43	0.82	0.84	0.69
0.45	0.74	0.55	0.41	0.81	0.83	0.67
0.50	0.71	0.55	0.39	0.79	0.83	0.66

Table 3: Comparison between Classical Reliability and Adjustment Factor for System Reliability $(R_p(t))$ under Prior Information

Time	$\sum_{i=1}^{n} x_i = 9.64, \quad \theta_0 = 1, \lambda = 1$			$\sum_{i=1}^{n} x_i = 21.33, \quad \theta_0 = 2, \quad \lambda = 1$		
t	CSR^{**}	$AFSR^{**}$	$CSR^{**} \times AFSR^{**}$	CSR^{**}	$AFSR^{**}$	$CSR^{**} \times AFSR^{**}$
0.01	0.9872	0.5102	0.5037	0.9937	0.5118	0.5086
0.05	0.9379	0.5053	0.4739	0.9690	0.5096	0.4938
0.10	0.8798	0.4992	0.4392	0.9390	0.5068	0.4759
0.15	0.8255	0.4931	0.4070	0.9100	0.5040	0.4586
0.20	0.7746	0.4871	0.3773	0.8819	0.5012	0.4420
0.25	0.7269	0.4810	0.3497	0.8547	0.4984	0.4260
0.30	0.6822	0.4750	0.3241	0.8284	0.4956	0.4105
0.35	0.6404	0.4690	0.3004	0.8029	0.4928	0.3957
0.40	0.6012	0.4631	0.2784	0.7782	0.4900	0.3813
0.45	0.5645	0.4572	0.2581	0.7543	0.4872	0.3675
0.50	0.5300	0.4512	0.2392	0.7311	0.4845	0.3542

Table 4: Comparison between Classical Reliability and Adjustment Factor for System Reliability $(R(t))$ under Posterior Information

Time	$\sum_{i=1}^{n} x_i = 9.64, \quad \theta_0 = 1, \quad \lambda = 1$			$\sum_{i=1}^{n} x_i = 21.33, \quad \theta_0 = 2, \lambda = 1$		
t	CSR_s^{**}	$AFSR_s^{**}$	$CSR_s^{**} \times AFSR_s^{**}$	CSR_s^{**}	$AFSR_s^{**}$	$CSR_s^{**} \times AFSR_s^{**}$
0.01	0.8687	0.4980	0.4326	0.9332	0.5062	0.4724
0.05	0.4978	0.4453	0.2217	0.7086	0.4817	0.3414
0.10	0.2513	0.3830	0.0962	0.5038	0.4516	0.2275
0.15	0.1284	0.3254	0.0417	0.359	0.4224	0.1517
0.20	0.0663	0.2734	0.0181	0.2569	0.3939	0.1012
0.25	0.0345	0.2271	0.0078	0.1841	0.3664	0.0675
0.30	0.0181	0.1868	0.0033	0.1323	0.3400	0.0449
0.35	0.0095	0.1521	0.0014	0.0953	0.3147	0.0299
0.40	0.0050	0.1227	0.0006	0.0687	0.2905	0.0199
0.45	0.0027	0.0981	0.0003	0.0497	0.2676	0.0133
0.50	0.0014	0.0778	0.0001	0.0360	0.2459	0.0088

Table 5: Comparison between Classical Reliability and Adjustment Factor for System Reliability $(R_S(t))$ under Posterior Information

Time	$\sum_{i=1}^{n} x_i = 9.64,\ \theta_0 = 1,\ \lambda = 1$			$\sum_{i=1}^{n} x_i = 21.33,\ \theta_0 = 2,\ \lambda = 1$		
t	CSR_p^{**}	$AFSR_p^{**}$	$CSR_p^{**} \times AFSR_p^{**}$	CSR_p^{**}	$AFSR_p^{**}$	$CSR_p^{**} \times AFSR_p^{**}$
0.01	0.9872	0.5102	0.5037	0.9937	0.5118	0.5086
0.05	0.9379	0.5053	0.4739	0.9690	0.5096	0.4938
0.10	0.8798	0.4992	0.4392	0.9390	0.5068	0.4759
0.15	0.8254	0.4931	0.4071	0.9100	0.5040	0.4586
0.20	0.7745	0.4871	0.3772	0.8812	0.5012	0.4420
0.25	0.7269	0.4810	0.3496	0.8547	0.4984	0.4260
0.30	0.6822	0.4750	0.3240	0.8284	0.4956	0.4105
0.35	0.6403	0.4690	0.3003	0.8029	0.4928	0.3957
0.40	0.6010	0.4630	0.2783	0.7782	0.4900	0.3813
0.45	0.5642	0.4570	0.2578	0.7542	0.4872	0.3675
0.50	0.5295	0.4510	0.2388	0.7311	0.4845	0.3542

Table 6: Comparison between Classical reliability and Adjustment Factor for System Reliability $\left(R_p(t)\right)$ under Posterior Information

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