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ANALYSIS OF THE PARITY PROGRESSION RATIOS

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Abstract
With changing reproductive patterns of the population overtime, it has become rather more important to know the process of family building as to how many women are moving from the lower parity to higher parity and in how much time the pattern of fertility advancement from i^{th} parity to $(i+1)^{th}$ is dealt through Parity Progression Ratios. During the recent years, many authors have studied the probability models in fertility particularly in parity progression ratios involving stochastic processes and renewal theory. Such studies have played important roles in fertility analysis. In the present study, an attempt has been made to work-out the distribution of maternal age over the reproductive span of woman and the probability distribution for woman proceeding to next higher parity. Finally, a life table on the parity progression has been constructed.

Keywords: Fertility analysis, parity progression ratio, birth interval distributions, parity specific fertility.

1. Introduction

In the past few decades considerable interest has been generated on the reproductive behaviour of the women over her reproductive span, which may be of immense use for future projection planning. Feeney (1983) developed a parity progression model to compute future population that resulted from the operation of a given set of parity progression ratios and birth interval distributions on an initial series of births distributed by order. Feeney has described the parity progression ratios and birth interval distributions as cohort groups of women who had a birth of a given order during a given period.

In the proposed study, an attempt has been made to look into the phenomenon of parity progression ratios by using the method of Pandey and Suchindran (1995) while taking the probability of ever bearing 'i' children (F_i) by the woman as the function of completed fertility rate over the reproductive age range (α, β). Thereafter, obtained recurrence relation between F_i and F_{i+1} to evaluate parity progression ratio (p_i) as F_{i+1}/F_i. Finally a life table on the parity progression based on Ram and Pathak (1989) has also been constructed and a comparison with the present study has been shown.

2. Materials and Methods

In the present study, the data on age-specific fertility rates of family planning door – to – door survey conducted in 1999 at Delhi under Family Planning Association of India has been undertaken to measure the probability of ever bearing child by women, life table on the parity progression and parity progression rates under different parities has been constructed.

Let F_i be the probability of ever bearing i children by the women in the reproductive age group (α, β) and also let m(x) be the instantaneous birth rate to a woman of age x to have a birth in the age group (x, x+dx) with m(x) = 0 for x < α and x > β.

The completed fertility rate (CFR) is given as

$$CFR = \int_{\alpha}^{\beta} m\,(a)\,da$$

and the cumulative fertility rate upto age x , say, CFR (x) as :

$$CFR\,(x) = \int_{\alpha}^{x} m\,(a)\,da$$

Let $f_1(x)dx$ be the instantaneous probability of experiencing a birth (the first birth) in the age interval (x , x + dx), then

$$f_1\,(x) = m\,(x)\,e^{-\int_{\alpha}^{x} m\,(a)\,da} \quad ; \quad \alpha \le x < \beta \qquad (1)$$

and the probability (F_1) that a woman would ever become a mother, is given by

$$F_1 = \int_{\alpha}^{\beta} f_1\,(x)\,dx = 1 - e^{-CFR}$$

if $f_2(y)$ dy be the probability of second birth in the age-interval (y, y+dy) such that $\alpha \le x < y$, then

$$f_2\,(y) = \int_{\alpha}^{y} \{\, m\,(y)\,e^{-\int_{\alpha}^{y} m\,(a)\,da} \,\}\, f_1\,(x)\,dx$$

on substitution $f_1(x)$ from (1) and after simplification, we get

$$f_2\,(y) = m\,(y)\,e^{CFR\,(y)}\,[CFR\,(y)]$$

Then the probability (F_2) that the women would ever become mother of two children, is given by

$$F_2 = \int_{\alpha}^{\beta} f_2\,(y)\,dy = 1 - e^{-CFR} \sum_{j=0}^{1} \frac{(CFR)^j}{j!}$$

In general, if $f_i\,(t)dt$ be the probability that the woman bears her i^{th} child (i ≥1) in the age interval (t, t+dt), it is given by

$$f_i\,(t) = \int_{\alpha}^{t} \{m\,(t)\,e^{-\int_{\alpha}^{t} m\,(a)\,da}\}\, f_{i-1}\,(z)\,dz \;;\; \alpha \le z < t < \beta$$

where $f_{i-1}(z)$ dz be the probability that the woman bears her $(i-1)^{th}$ child in the age-interval (z, z+dz). By induction,

$$f_i\,(t) = \int_{\alpha}^{t} \{m\,(t)\,e^{-\int_{\alpha}^{t} m\,(a)\,da}\}\, \frac{[CFR\,(z)]^{i-2}}{(i-2)!}\, e^{-CFR\,(Z)}\, m\,(z)\,dz$$

$$= m(t)\, e^{-CFR(t)} \frac{[CFR(z)]^{i-1}}{(i-1)!}$$

The probability of ever bearing 'i' children (F_i) by the woman is given by

$$F_i = \int_{\alpha}^{\beta} f_i(t)\,dt = 1 - e^{-CFR} \sum_{j=0}^{i-1} \frac{(CFR)^j}{j!}$$ for different values of i and j

In particular, different F_i's can be evaluated as follows:

Let F_1 = probability of ever bearing 1 child (F_1) by the women over the reproductive span (α,β) is

$$\text{Then } F_1 = 1 - e^{-CFR}$$

Similarly probability for ever bearing 2,3,4…….. so on, children are given by

$$F_2 = 1 - e^{-CFR}(1 + CFR)$$

$$F_3 = 1 - e^{-CFR}\left[1 + CFR + \frac{(CFR)^2}{2!}\right]$$

$$F_n = 1 - e^{-CFR}\left[1 + CFR + \frac{(CFR)^2}{2!} + \ldots\ldots\ldots + \frac{(CFR)^{n-1}}{(n-1)!}\right]$$

Let p_i, $i \ge 0$, be the probability that a woman of parity 'i' would be proceeding for the next higher parity (i+1), can be estimated as $P_i = F_{i+1} / F_i$;; $P_1 = F_2 / F_1$; $P_2 = F_3 / F_2$; …………. and in general, $P_n = F_{n+1} / F_n$.

3. Results and Discussions

Different values of F_i's and P_i's have been computed as shown in Tables 1 and 2 while distribution of women in different parities with respect to age at the time of birth of the child is depicted in Table 3. Further probability of not going to the next parity till the end of reproductive span is also given in Table 4. Based on the data of Table 4, a life table on the parity progression has been constructed and shown below in Table 5.

Age Group	F1	F2	F3	F4	F5	F6	F7
15-19	0.0374487	0.0007101	9.006E-06	8.577E-08	6.539E-10	4.156E-12	2.265E-14
20-24	0.2587855	0.0368175	0.0035817	0.000264	1.565E-05	7.755E-07	3.299E-08
25-29	0.2624825	0.0379339	0.0037502	0.000281	1.693E-05	8.528E-07	3.689E-08
30-34	0.1903517	0.0193902	0.0013404	7E-05	2.935E-06	1.028E-07	3.088E-09
35-39	0.1185810	0.0073264	0.0003050	9.562E-06	2.404E-07	5.041E-09	9.07E-11
40-44	0.0722565	0.0026758	6.647E-05	1.242E-06	1.858E-08	2.318E-10	2.48E-12
45-49	0.0668958	0.0022892	5.253E-05	9.06E-07	1.252E-08	1.442E-10	1.425E-12

Table 1: Probability of ever bearing 'i' children (F_i) at different age span of women

Age Group	P1 (F2/F1)	P2 (F3/F2)	P3 (F4/F3)	P4 (F5/F4)	P5 (F6/F5)	P6 (F7/F6)
15-19	0.0189626	0.0126821	0.0095237	0.0076238	0.0063555	0.0054499
20-24	0.1422705	0.0972822	0.0737173	0.0592797	0.049546	0.0425465
25-29	0.1445197	0.0988627	0.0749282	0.0602588	0.0503669	0.0432528
30-34	0.1018649	0.0691295	0.0522217	0.0419284	0.0350125	0.0300494
35-39	0.0617838	0.0416278	0.0313543	0.0251371	0.0209731	0.0179907
40-44	0.0370313	0.024843	0.0186792	0.0149623	0.0124775	0.0106999
Probability upto age 44 years (= 1- Prob.(15-44))						
	0.9657803	**0.9770543**	**0.9827507**	**0.9861845**	**0.9884794**	**0.9901217**
45-49	0.0342197	0.0229457	0.0172493	0.0138155	0.0115206	0.0098783

Table 2: Probability of woman parity 'i' proceeding age span wise for next higher parity

	Parity 1		Parity 2		Parity 3		Parity 4		Parity 5		Parity 6	
Age Group	Wo-men	Propor-tion	Wo-men	Propor-tion	Wo-men	Propor-tion	Wo-men	Propor-tion	Wo-men	Propor-tion	Wo-men	Propor-tion
15-19	5	0.986842	1	0.98781	1	0.990099	-	0.972973	-	0.94444	-	0.9
20-24	46		24		6		-		-		-	
25-29	17		37		33		6		1		1	
30-34	5		15		35		13		11		3	
35-39	1		2		18		14		3		2	
40-44	1		2		7		3		2		3	
45-49	1	0.01315	1	0.01219	1	0.00990	1	0.02702	1	0.05556	1	0.1
Total	**76**		82		101		37		18		10	

Table 3: Distribution of women in different parities with respect to age at the time of the birth of the child.

Parity	Probability of not going to the parity (i+1) till the end of reproductive span
1	1 - (0.986842*0.9657803 + 0.013158*0.0342197) = 0.046477
2	1 - (0.987805*0.9770543 + 0.012195*0.0229457) = 0.034581
3	1 - (0.99099*0.9827507+ 0.009901*0.0172493) = 0.025933
4	1 - (0.972973*0.9861845 + 0.027027*0.0138155) = 0.040096
5	1 - (0.94444*0.9884794 + 0.05556*0.0115206) = 0.065801
6	1 - (0.9*0.9901217+ 0.1*0.0098783) = 0.107903

Table 4 : Probability of not going to the next parity till the end of the reproductive span.

Where x = parity number.

lx = number of cohorts delivering children at parity x.

dx = number of delivered children in each parity.

qx = probability of not going to the next parity.
px = probability of advancing to the next parity.
Lx= estimated total time lived by cohort delivering children between x to x+1 parity.
Tx = total number of years lived by cohort from parity x onwards.
e^0x = expected number of days for completion of delivery period at the x^{th} parity.

Finally a comparison between the parity progression rates of the present study with that of Ram and Pathak (1989) is presented in Table 6. Ram and Pathak have worked out the parity progression ratios, using

$$P_i = \frac{(X_w - X_i)\, r_i}{1 + (X_w - X_{i+1})\, r_i}$$

where X_w = maximum age of reproduction
X_i = mean age of women at their i^{th} birth
r_i = parity specific fertility rate

Parity	Ram-Pathak	Present studies
1-2	0.8195	0.5407
2-3	0.8587	0.3450
3-4	0.8387	0.2777
4-5	0.4578	0.2230
5-6	0.3011	0.1417
6-7	0.1597	0.1599

Table 6: A comparison of parity progression rates

The comparison shows that our estimates are improvement over Ram and Pathak's estimates in the present study. The difference may be attributed towards the differences in intrinsic behaviour of fertility in different populations taken into consideration.

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