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A HYBRID GROUP ACCEPTANCE SAMPLING PLANS FOR LIFETIMES BASED ON LOG-LOGISTIC DISTRIBUTION

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Abstract
In this paper, a hybrid group acceptance sampling plan is developed for a truncated life test when the lifetime of an item follows log-logistic distribution. The minimum number of testers and acceptance number are determined when the consumer's risk and the test termination time and group size are specified. The operating characteristic values according to various quality levels are found and the minimum ratios of the true average life to the specified life at the specified producer's risk are obtained. The results are explained with examples.

Key words: Log-logistic distribution; Group acceptance sampling; Consumer's risk; Operating characteristics; Producer's risk; Truncated life test.

Mathematics Subject Classification: 62N05; 62P30.

1. Introduction
An acceptance sampling plan is a scheme that establishes the minimum sample size to be used for testing. This becomes particularly important if the quality of product is defined by its lifetime. Often, it is implicitly assumed when designing a sampling plan that only a single item is put in a tester. However, in practice testers accommodating a multiple number of items at a time are used because testing time and cost can be saved by testing items simultaneously. The items in a tester can be regarded as a group and the number of items in a group is called the group size. An acceptance sampling plan based on such groups of items is called a group acceptance sampling plan (GASP). The method of determining the minimum number of items for a predetermined number of groups is called as hybrid group acceptance sampling plan (HGASP). The minimum number of items in each group is very important to save the time and cost. Moreover, if the group size (r) is very large then taking a decision very difficult. This HGASP we fix group size (r) in our hands and also HGASP more useful than ordinary GASP, instead of fixing the group size (r), we can fix the number of groups at our convenience. If the HGASP is used in conjunction with truncated life tests, it is called a HGASP based on truncated life test assuming that the lifetime of product follows a certain probability distribution. For such a type of test, the determination of the sample size is equivalent to determine the number of testers. This type of testers is frequently used in sudden death testing. The sudden death tests are discussed by Pascual and Meeker (1998) and Vlcek *et al.* (2003). Recently, Jun *et al.* (2006) proposed the sudden death test under the assumption that the lifetime of items follows the Weibull distribution with known shape parameter. They developed the single and double group acceptance sampling plans in sudden death testing. More recently, Aslam and Jun (2009) for inverse Rayleigh and log-logistic distributions,

Srinivasa Rao (2009) for generalized exponential distribution and Srinivasa Rao (2010) for Marshall-Olkin extended Lomax distribution are proposed the group acceptance sampling plan based on truncated life test.

Acceptance sampling based on truncated life tests having single-item group for a variety of distributions were discussed by Epstein (1954), Sobel and Tischendrof (1959), Goode and Kao (1961), Gupta and Groll (1961), Gupta (1962), Fertig and Mann (1980), Kantam and Rosaiah (1998), Kantam *et al.* (2001), Baklizi (2003), Baklizi and El Masri (2004), Rosaiah and Kantam (2005), Rosaiah *et al.* (2006, 2007, 2007), Tsai and Wu (2006), Balakrishnan *et al.* (2007), Aslam (2007), Aslam and Shahbaz (2007), Aslam and Kantam (2008) and Srinivasa Rao *et al*. (2008, 2009).

The purpose of this paper is to propose a HGASP based on truncated life tests when the lifetime of a product follows the two-parameter log-logistic distribution. For an excellent review of this distribution, the readers are referred to Johnson *et al.* (1995). Let T be a lifetime that is distributed according to log-logistic distribution with two parameters $\sigma > 0$ and $\delta > 1$. The probability density function (p.d.f.) and cumulative distribution function (c.d.f) of the two-parameter log-logistic distribution respectively, are given by

$$g(t;\sigma,\delta) = \frac{\delta}{\sigma} \frac{(t/\sigma)^{\delta-1}}{\left[1+(t/\sigma)^{\delta}\right]^2}; \quad \text{for } t \geq 0, \sigma > 0, \delta > 1 \qquad (1)$$

$$G_T(t;\sigma,\delta) = \frac{(t/\sigma)^{\delta}}{[1+(t/\sigma)^{\delta}]}; \quad \text{for t} \geq 0, \sigma > 0, \delta > 1. \qquad (2)$$

where σ and δ are scale and shape parameters respectively. The median of this distribution for $\delta = 2$ is given by $m = \sigma$. The decision to acceptance of lot can be related to a hypothesis testing. The null hypothesis is “lot median is greater than or equal to a specified quantity” and the alternative hypothesis is “lot median is smaller than a specified quantity.” On the basis of the observed number of failures in a sample, if the null hypothesis has failed to reject, then the lot is accepted as a good lot, which will ensure a certain quality of the products. It is important to note that a log-logistic distribution is a skewed one, therefore it is preferable to use the median life to develop acceptance plans rather than the mean life. Kantam *et al.* (2001) developed single acceptance sampling plans based on the mean of the log-logistic distribution. In Section 2, we describe the proposed HGASP. The operating characteristics values in Section 3. The results are explained with some examples in Section 4, and finally, some conclusions are given in Section 5.

2. The Hybrid Group Acceptance Sampling Plan (HGASP)

Let m represent the true median life of a product and m_0 denote the specified median life of an item, under the assumption that the lifetime of an item follows log-logistic distributions. A product is considered as good and accepted for consumer’s use if the sample information supports the hypothesis $H_0 : m \geq m_0$. On the other hand, the lot of the product is rejected. In acceptance sampling schemes, this hypothesis is tested based on the number of failures from a sample in a pre-fixed time. If the number of failures exceeds the action limit *c* we reject the lot. We will accept the lot if there is enough evidence that $m \geq m_0$ at certain level of consumer's risk. Otherwise, we reject the lot. Let us propose the following HGASP based on the truncated life test:

1. Determine the number of testers, *r* and assign the *r* items to each predefined *g*, groups, the required sample size for a lot is *n*= *r g*.
2. Pre-fix the acceptance number, *c* for each group and the experiment time t_0.
3. Accept the lot if at most c failures occur in each of all groups.
4. Terminate the experiment if more than *c* failures occur in any group and reject the lot.

The proposed sampling plan is an extension of the ordinary sampling plan available in literature such as in Kantam *et al.* (2001), for *r* =1. We are interested in determining the number of tester's *r*, required for log-logistic distributions and various values of acceptance number *c*, whereas the number of groups *g*, and the termination time t_0 are assumed to be specified. Since it is convenient to set the termination time as a multiple of the specified value $m_0 = \sigma_0$ of the median, we will consider $t_0 = a\sigma_0$ for a given constant *a* (termination ratio).

The probability (α) of rejecting a good lot is called the producer's risk, whereas the probability (β) of accepting a bad lot is known as the consumer's risk. The parameter value *r* of the proposed sampling plan is determined for ensuring the consumer's risk β. Often, the consumer's risk β is expressed by the consumer's confidence level. If the confidence level is p^*, then the consumer's risk will be $\beta = 1 - p^*$. We will determine the number of tester's *r* in the proposed sampling plan so that the consumer's risk does not exceed *a* given value β. If the lot size is large enough, we can use the binomial distribution to develop the HGASP. According to the HGASP the lot of products is accepted only if there are at most *c* failures observed in each of the *g*, groups. The HGASP is characterized by the three parameters $(n, c, t/\sigma_0)$. The lot acceptance probability is

$$L(p) = \left[\sum_{i=0}^{c} \binom{r}{i} p^i (1-p)^{r-i} \right]^g \qquad (3)$$

where *p* is the probability that an item in a tester fails before the termination time $t_0 = a\sigma_0$. The probability *p* for the log-logistic distributions with $\delta = 2$ is given by

$$p = G_T(t_0) = \frac{\{a/(\sigma/\sigma_0)\}^\delta}{1+\{a/(\sigma/\sigma_0)\}^\delta} . \qquad (4)$$

The minimum number of testers required can be determined by considering the consumer's risk when the true median life equals the specified median life ($\sigma = \sigma_0$) through the following inequality:

$$L(p_0) \le \beta \tag{5}$$

where p_0 is the failure probability at $\sigma = \sigma_0$, and it is given by

$$p_0 = \frac{a^\delta}{1 + a^\delta}. \tag{6}$$

Table 1 shows for the pre-fix consumer's risk, number of groups, acceptance number and truncation time to obtain the minimum testers. The minimum number of testers required for the proposed sampling plan in case of the log-logistic distributions for the special case $\delta = 2$ are calculated and displayed in Table 2. The used values of the consumer's risk, the group size, the acceptance number and the time multiplier are given in Table 1.

Table 1: Consumer's risk (β), truncated time (*a*), group size (*g*) and acceptance number (*c*)

β	0.25 0.10 0.05 0.01
a	0.7 0.8 1.0 1.2 1.5 2.0
g	2 3 4 5 6 7 8 9 10
c	0 1 2 3 4 5 6 7 8

<<<<< Table 2 around here >>>>>

3. Operating Characteristics

The probability of acceptance can be regarded as a function of the deviation of specified median from the true median. This function is called operating characteristic (OC) function of the sampling plan. Once the minimum number of testers is obtained, one may be interested to find the probability of acceptance of a lot when the quality (or reliability) of the product is good enough. As mentioned earlier, the product is considered to be good if $\sigma > \sigma_0$ or $\frac{\sigma}{\sigma_0} > 1$. For $\delta = 2$ the probabilities of acceptance are displayed in Table 3 based on (3) for various values of the median ratios σ/σ_0, producer's risks β, and time multiplier *a* that are given in Table 1. From Table 3 we see that OC values increase more quickly as the median ratio increases. For example, when β = 0.25, *g*=4, *c*=2 and *a*= 0.7, the number of testers required is *r*=6. However, if the true median lifetime is twice the specified median lifetime ($\sigma/\sigma_0 = 2$) the producer's risk is approximately α =1- 0.9218=0.0782, while α is 0.002 when the true value of median is 4 times the specified one.

The producer may be interested in enhancing the quality level of the product so that the acceptance probability should be greater than a specified level. At the producer's risk α the minimum ratio σ/σ_0 can be obtained by satisfying the following inequality:

$$\left[\sum_{i=0}^{c}\binom{r}{i} p^i (1-p)^{r-i}\right]^g \geq 1-\alpha \tag{7}$$

where p is given by (4) and r is chosen at the consumer's risk β when $\sigma/\sigma_0 = 1$. Table 4 shows the minimum ratio of σ/σ_0 for log-logistic distributions with $\delta = 2$ at the producer's risk of α = 0.05 under the plan parameters given in Table 1. Table 4 shows that for fixed values of *g* and *c*, the median ratio increased as the termination ratio increased. For example, when β =0.25, *r*=6 *g*=4, *c*=2 and *a*=0.7, for obtaining a producer's risk α =0.05 increase the true value σ of median to 2.20 times the specified value σ_0 is required.

4. Tables And Examples

The design parameters of HGASP are found at the various values of the consumer's risk and the test termination time multiplier in Table 2. It should be noted that if one needs the minimum sample size, it can be obtained by $n = r \times g$. Table 2 indicates that, as the test termination time multiplier *a* increases, the number of testers *r* decrease, i.e., a smaller number of testers is needed, if the test termination time multiplier increases at a fixed number of groups. For an example, from Table 2, if β =0.10, *g*=4, c=2 and a changes from 0.7 to 0.8, the required values of design parameters of HGASP have been changed from *r* =8 to *r* =7. However, this trend is not monotonic since it depends on the acceptance number as well. The probability of acceptance for the lot at the median ratio corresponding to the producer's risk is also given in Table 3. Finally, Table 4 presents the minimum ratios of true median to the specified median for the acceptance of a lot with producer's risk α = 0.05 for given parameter values.

Suppose that the lifetime of a product follows the log-logistic distributions with $\delta = 2$. It is desired to design a HGASP to test if the median is greater than 1,000 hours based on a testing time of 700 hours and using 4 groups. It is assumed that c=2 and β =0.10. This leads to the termination multiplier *a* =0.700. From Table 2 the minimum number of testers required is *r* =8. Thus, we will draw a random sample of size 32 items and allocate 8 items to each of 4 groups to put on test for 700 hours. This indicates that a total of 32 products are needed and that 8 items are allocated to each of 4 testers. We will accept the lot if no more than 2 failure occurs before 700 hours in each of 4 groups. We truncate the experiment as soon as the 3rd failure occurs before the 700th hours. For this proposed sampling plan the probability of acceptance is 0.8223 when the true mean is 2,000 hours. This shows that, if the true median life is 2 times of 1000 hours, the producer's risk is 0.1777. If we need the ratio corresponding to the producer's risk of 0.05, we can obtain it from Table 4. For example, when β =0.10, r=8, g=4, c=2, *a*= 0.700, the ratios of σ/σ_0 is 2.63.

5. Conclusion

In this paper, a hybrid group acceptance sampling plan from the truncated life test was proposed, the number of testers and the acceptance number was determined for log-logistic distributions with $\delta = 2$ when the consumer's risk (β) and the other plan parameters are specified. It can be observed that the minimum number of testers required is decreases as test termination time multiplier increases and also the operating characteristics values increases more rapidly as the quality improves. This HGASP can be used when a multiple number of items at a time are adopted for a life test and it would be beneficial in terms of test time and cost because a group of items will be tested simultaneously.

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Table 2: Minimum number of testers (*r*) and acceptance number (*c*) for the proposed plan for the log-logistic distributions with $\delta = 2$.

β	*g*	*c*	*a*					
			0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	0	2	2	1	1	1	1
0.25	3	1	4	4	3	3	2	2
0.25	4	2	6	5	4	4	3	3
0.25	5	3	8	7	6	5	5	4
0.25	6	4	11	9	7	6	6	5
0.25	7	5	13	11	9	8	7	6
0.25	8	6	15	13	10	9	8	7
0.25	9	7	17	15	12	10	9	8
0.25	10	8	19	17	13	12	10	9
0.10	2	0	3	3	2	2	1	1
0.10	3	1	6	5	4	3	3	2
0.10	4	2	8	7	5	4	4	3
0.10	5	3	10	8	7	6	5	4
0.10	6	4	12	10	8	7	6	5
0.10	7	5	14	12	10	8	7	7
0.10	8	6	17	14	11	10	9	8
0.10	9	7	19	16	13	11	10	9
0.10	10	8	21	18	14	13	11	10
0.05	2	0	4	4	3	2	2	1
0.05	3	1	6	5	4	3	3	2
0.05	4	2	9	7	6	5	4	4
0.05	5	3	11	9	7	6	5	5
0.05	6	4	13	11	9	7	6	6
0.05	7	5	15	13	10	9	8	7
0.05	8	6	18	15	12	10	9	8
0.05	9	7	20	17	13	12	10	9
0.05	10	8	22	19	15	13	11	10
0.01	2	0	6	5	4	3	2	2
0.01	3	1	8	7	5	4	4	3
0.01	4	2	10	9	7	6	5	4
0.01	5	3	13	11	8	7	6	5
0.01	6	4	15	13	10	8	7	6
0.01	7	5	17	14	11	10	8	7
0.01	8	6	19	16	13	11	9	8
0.01	9	7	22	18	15	12	11	9
0.01	10	8	24	20	16	14	12	10

Table 3: Operating characteristics values of the hybrid group sampling plan with *g*=4 and *c* =2 for log-logistic distributions with $\delta = 2$.

β	r	a	σ/σ_0					
			2	4	6	8	10	12
0.25	6	0.7	0.9218	0.9980	0.9998	1.0000	1.0000	1.0000
0.25	5	0.8	0.9182	0.9979	0.9998	1.0000	1.0000	1.0000
0.25	4	1.0	0.8956	0.9969	0.9997	0.9999	1.0000	1.0000
0.25	4	1.2	0.7825	0.9916	0.9991	0.9998	1.0000	1.0000
0.25	3	1.5	0.8260	0.9925	0.9992	0.9998	1.0000	1.0000
0.25	3	2.0	0.5862	0.9684	0.9960	0.9992	0.9998	0.9999
0.10	8	0.7	0.8223	0.9948	0.9995	0.9999	1.0000	1.0000
0.10	7	0.8	0.7819	0.9929	0.9993	0.9999	1.0000	1.0000
0.10	5	1.0	0.7877	0.9926	0.9992	0.9999	1.0000	1.0000
0.10	4	1.2	0.7825	0.9916	0.9991	0.9998	1.0000	1.0000
0.10	4	1.5	0.5566	0.9731	0.9969	0.9994	0.9998	0.9999
0.10	3	2.0	0.5862	0.9684	0.9960	0.9992	0.9998	0.9999
0.05	9	0.7	0.7612	0.9923	0.9992	0.9999	1.0000	1.0000
0.05	7	0.8	0.7819	0.9929	0.9993	0.9999	1.0000	1.0000
0.05	6	1.0	0.6594	0.9858	0.9985	0.9997	0.9999	1.0000
0.05	5	1.2	0.6007	0.9803	0.9979	0.9996	0.9999	1.0000
0.05	4	1.5	0.5566	0.9731	0.9969	0.9994	0.9998	0.9999
0.05	4	2.0	0.2234	0.8956	0.9853	0.9969	0.9991	0.9997
0.01	10	0.7	0.6955	0.9893	0.9989	0.9998	0.9999	1.0000
0.01	9	0.8	0.6105	0.9840	0.9983	0.9997	0.9999	1.0000
0.01	7	1.0	0.5269	0.9764	0.9975	0.9995	0.9999	1.0000
0.01	6	1.2	0.4234	0.9633	0.9958	0.9992	0.9998	0.9999
0.01	5	1.5	0.3149	0.9396	0.9926	0.9985	0.9996	0.9999
0.01	4	2.0	0.2234	0.8956	0.9853	0.9969	0.9991	0.9997

Table 4: Minimum ratio of the values of true median and specified median for the producer's risk of α = 0.05 in the case of log-logistic distributions with $\delta = 2$.

B	g	c	a					
			0.7	0.8	1.0	1.2	1.5	2.0
0.25	2	0	6.16	7.04	6.21	7.45	9.31	12.42
0.25	3	1	2.90	3.32	3.46	4.15	3.89	5.17
0.25	4	2	2.20	2.22	2.36	2.82	2.72	3.62
0.25	5	3	1.88	1.96	2.18	2.24	2.80	2.93
0.25	6	4	1.82	1.80	1.84	1.91	2.38	2.53
0.25	7	5	1.67	1.70	1.80	1.94	2.11	2.26
0.25	8	6	1.57	1.62	1.62	1.75	1.91	2.06
0.25	9	7	1.50	1.56	1.62	1.61	1.76	1.91
0.25	10	8	1.43	1.51	1.49	1.65	1.64	1.79
0.10	2	0	7.56	8.63	8.81	10.57	9.31	12.42
0.10	3	1	3.67	3.78	4.15	4.15	5.20	5.17
0.10	4	2	2.63	2.76	2.77	2.82	3.53	3.62
0.10	5	3	2.18	2.15	2.45	2.62	2.80	2.93
0.10	6	4	1.93	1.94	2.06	2.21	2.38	2.53
0.10	7	5	1.76	1.81	1.97	1.94	2.11	2.81
0.10	8	6	1.71	1.71	1.77	1.95	2.19	2.54
0.10	9	7	1.62	1.64	1.73	1.78	2.01	2.34
0.10	10	8	1.54	1.58	1.60	1.79	1.87	2.18
0.05	2	0	8.73	9.98	10.80	10.57	13.22	12.42
0.05	3	1	3.67	3.78	4.15	4.15	5.20	5.17
0.05	4	2	2.82	2.76	3.14	3.33	3.53	4.71
0.05	5	3	2.31	2.33	2.45	2.62	2.80	3.74
0.05	6	4	2.03	2.08	2.26	2.21	2.38	3.18
0.05	7	5	1.84	1.91	1.97	2.17	2.43	2.81
0.05	8	6	1.78	1.80	1.90	1.95	2.19	2.54
0.05	9	7	1.67	1.71	1.73	1.94	2.01	2.34
0.05	10	8	1.59	1.64	1.70	1.79	1.87	2.18
0.01	2	0	10.70	11.16	12.47	12.95	13.22	17.62
0.01	3	1	4.29	4.57	4.74	4.97	6.23	6.95
0.01	4	2	2.99	3.23	3.46	3.78	4.17	4.71
0.01	5	3	2.55	2.64	2.69	2.94	3.27	3.74
0.01	6	4	2.22	2.32	2.43	2.47	2.76	3.18
0.01	7	5	2.00	2.01	2.12	2.36	2.43	2.81
0.01	8	6	1.84	1.88	2.02	2.12	2.19	2.54
0.01	9	7	1.78	1.78	1.95	1.94	2.23	2.34
0.01	10	8	1.69	1.70	1.80	1.92	2.07	2.18