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GENERAL ANALYSIS OF SINGLE PRODUCT INVENTORY MODEL – WITH PRODUCTION BY TWO UNITS SYSTEM AND SALES

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Abstract

In this paper a two units system produces a single product for sale. The single product is produced one by one. The sale time starts when k number of products is produced or when the two units system fails whichever occurs first. Assuming the production and sale time of products have general distribution and the failure and repair rates of the two units are constants, the double Laplace transform of the joint distribution function of the time to sale and sale time are obtained. Their expectations are derived. Numerical examples are presented.

Mathematics Subject Classification: 30c45, 30c80.

Key Words: Inventory System, Single Product, Two-Unit System, Seasonal Sales.

1. Introduction

Several researchers studied single commodity inventory systems of (s,S) type. Arrow, Karlin and Scart [1] first analysed such inventory systems. Daniel and Ramanarayanan [2] discussed (s, S) inventory system with random lead times and unit demand models. Murthy and Ramanarayanan [3, 4, 5,] have considered several (s,S) inventory systems. Ramanarayanan [6,7] have introduced, “General Analysis 1-out of 2: F Systems exposed to cumulative damage processes”.,and General Analysis of of Systems in which a 2-unit system is a subsystem. Usha, Eswariprem, Ramanarayanan[8,9,10]have produced some results in Stochastic Analysis of Time to Vital Organs Failure of Gestational Diabetic Person. Wu and Liang-Yuh ouyang [11] studied (Q, R, L), at inventory model with defective items. In real life inventory models different types of single products are produced for sales by the companies. For example there is seasonal demand for ice-cream in summer and in textiles there is a seasonal demand for woollen clothes in winter season and so on. In this paper we consider an inventory system in which single product is produced one by one by a two-unit system. The sale time starts when k number of products are produced or two- unit system fails. In this paper Double Laplace Transform of distribution function of time to sale and sale time is derived. Their expectation times and numerical examples are presented.

2. Model

2.1 Assumptions

(i) The company produces single product only one type is produced. The production time of product is random variable with cdf $G(x)$.

(ii) The products are produced by a two unit system which fails when the two units are down and it works when at least one unit is good. Let the probability that either of two units fails during (t, t+Δt) given that the two units are operating at time t, be $\lambda_1\Delta t+o(\Delta t)$ and the probability that one fixed unit fails during (t,t+Δt) given that it is operating at time t, be $\lambda_2\Delta t+o(\Delta t)$. The repair rate of the failed unit is μ

(iii) Sale time starts when k numbers of products are produced or when the two- unit system fails.

(iv) The products are sold and the selling time of a product is random variable with cdf $R(y)$.

2.2 Notations

T – Time to sales

R – Sales time

λ_1 - The parameter of the exponential life time distribution when two units are good

λ_2 - The parameter of the exponential life time distribution when one is in failed state μ - The parameter of the exponential repair time distribution of the failed unit which is taken for repair.

g_n(x) is n-fold convolution of g with itself

r_i(y) - pdf of sales time for i number of production

r_k(y) - pdf of sales time for k number of production

$*$ - indicates Laplace Transform

$\bar{G}$(x) = 1-G(x)

2.3 Analysis

We note that the probability of n number of pairs produced in (o, t)

$$= \int_0^t g_n(x)\overline{G_X}(t-x)dx. \qquad (1)$$

Since the selling time starts when k numbers of products are produced or when the two unit system fails, T- the time to start sales is given by

T = min (time to produce k number of products, the time at which the two units system fails)

To find the distribution function of the time to failure of the two units system, we need the functions $P_{0,0}$(u)=P (at time u the two units system is working, the system does not fail during (0,u) | at time 0 the two units of the system are working). $P_{0,1}$(u)=P (at time u one unit is under repair, system does not fail during (0,u) | at time 0 the units are working). $P_{0,2}$(u) be the pdf of time to failure of the two units system, $P_{0,2}$(u)=p(the two –unit system fails during (u,u+du), it does not fail during (0,u) | the two units are working at time 0) .

We now calculate the P.,.(.) functions. $P_{0,0}$(x) satisfies the following

$$P_{0,0}(x) = e^{-\lambda_1 x} + \int_0^x \lambda_1 e^{-\lambda_1 u} P_{1,0}(x-u)du$$

where $P_{1,0}(x)$ = P(at time x the two units are working | at time 0 one unit is in failed state) $P_{1,0}(x) = \int_0^x \mu e^{-\mu u} e^{-\lambda_2 u}\, e^{-\lambda_1 (x-u)} du + \int_0^x \int_0^v \mu e^{-\mu u} e^{-\lambda_2 u} \lambda_1 e^{-\lambda_1 (v-u)}\, du P_{1,0}(x-v) dv$.

The first term is the probability that the failed unit is repaired at u and no unit of the system fails during (0, x) and the second term is the probability that the failed unit is repaired at u and other unit does not fail during (0,u), a unit fails at v>u and at x all the two units are working.

Laplace transforms of the above equations give

$P^*_{0,0}(s) = (\lambda_2+\mu+s) \,|\, [s^2+s(\lambda_1+\lambda_2+\mu)+ \lambda_1 \lambda_2]$ (2)

Where * indicates Laplace transform.

Using a similar argument we find

$P_{0,1}(x) = \int_0^x \lambda_1 e^{-\lambda_1 u} e^{-\mu(x-u)} e^{-\lambda_2 (x-u)} du + \int_0^x \int_0^v \lambda_1 e^{-\lambda_1 u}\, \mu\, e^{-\mu(v-u)} e^{-\lambda_2 (v-u)}\, du P_{0,1}(x-v) dv$.

Taking Laplace Transform $P^*_{0,1}(s) = \lambda_1/[s^2+s(\lambda_1+\lambda_2+\mu)+ \lambda_1 \lambda_2]$ (3)

The failure density $P_{0,2}(x)$ satisfies

$P_{0,2}(x) = \int_0^x \lambda_1 e^{-\lambda_1 u}\, P_{1,2}(x-u) du$

where $P_{1,2}(x)\, dx$ = P(the two unit system fails during (x,x+dx) | at time 0 one unit of the system is under repair):

$$P_{1,2}(x) = \lambda_2\, e^{-\lambda_2 x} e^{-\mu x} + \int_0^x e^{-\lambda_2 u} \mu e^{-\mu u}\, P_{0,2}(x-u) du$$

By Laplace transformation,

$$P^*_{0,2}(s) = \lambda_1 \lambda_2 \,|\, [s^2+s(\lambda_1+\lambda_2+\mu)+ \lambda_1 \lambda_2] \quad (4)$$

Equation (2) – (4) can be inverted easily

$P_{0,0}(t) = (\frac{1}{2}) e^{-at}[e^{bt} + e^{-bt}] + \left(\frac{1}{4b}\right)(\lambda_2 - \lambda_1 + \mu) e^{-at}[e^{bt} - e^{-bt}]$

$P_{0,1}(t) = (\frac{\lambda_1}{2b})\, e^{-at}[e^{bt} - e^{-bt}]$ λ_2

$P_{0,2}(t) = (\frac{\lambda_1 \lambda_2}{2b})\, e^{-at}[e^{bt} - e^{-bt}]$

where $a = (\lambda_1+\lambda_2+\mu)/2$

and $b = (\frac{1}{2}) \sqrt{(\lambda_1 - \lambda_2)^2 + (\mu)^2 + 2\mu(\lambda_1 + \lambda_2)}$

$P_{0,0}(t)+P_{0,1}(t)$ = survival function of two unit system.

$$= e^{-(a-b)t}[\frac{1}{2}+\frac{\lambda_2}{4b}-\frac{\lambda_1}{4b}+\frac{\mu}{4b}] + e^{-(a+b)t}[\frac{1}{2}-\frac{\lambda_2}{4b}+\frac{\lambda_1}{4b}-\frac{\mu}{4b}]$$

$$= c_1\, e^{-(a-b)t} + c_2\, e^{-(a+b)t}$$

Where $c_1 = [\frac{1}{2} + \frac{\lambda_2}{4b} - \frac{\lambda_1}{4b} + \frac{\mu}{4b}]$ and $c_2 = [\frac{1}{2} - \frac{\lambda_2}{4b} + \frac{\lambda_1}{4b} - \frac{\mu}{4b}]$, $c_1 + c_2 = 1$

$p_{0.2}(t)$ = pdf of two unit system$\frac{1}{2}$

$$= \frac{\lambda_1 \lambda_2}{2b}[e^{-(a-b)t} - e^{-(a+b)t}]$$

$$= c_3\, [e^{-(a-b)t} - e^{-(a+b)t}] \quad \text{where } c_3 = \frac{\lambda_1 \lambda_2}{2b}$$

The pdf of T is

$$f_T(t) = g_k(t)(c_1\, e^{-(a-b)t} + c_2\, e^{-(a+b)t}) + c_3\, (e^{-(a-b)t} - e^{-(a+b)t})\, [\sum_{i=0}^{k-1} \int_0^t g_i(x) \bar{G}(t-x) dx] \quad (5)$$

The first term of the right side of (5) is the part of the pdf that the time to produce k number of products is t and the two units system has not failed upto time t. The second term is part of the pdf that the two units system fails at time t, the time to produce i number of products is x, $0 \leq i \leq k-1$

This gives the joint pdf of time to start sales T and total sales time of single product R as follows:
Considering the sales time of the pairs.

$$f_{T,R}(x,y) = g_k(x)(c_1\, e^{-(a-b)x} + c_2\, e^{-(a+b)x})\, r_k(y) + c_3\, [e^{-(a-b)x} - e^{-(a+b)x}] \left[\sum_{i=0}^{k-1} \int_0^x g_i(x) \bar{G}(x-u)du\, r_i(y)\right] \qquad (6)$$

The double Laplace transform of the pdf is given by

$$f_{T,R}^*(\varepsilon, \eta) = \int_0^\infty \int_0^\infty e^{-\varepsilon x} e^{-\eta y} f_{T,R}(x,y) dxdy$$

$$f_{T,R}^*(\varepsilon, \eta) = \int_0^\infty \int_0^\infty e^{-\varepsilon x - \eta y} g_k(x)(c_1\, e^{-(a-b)x} + c_2\, e^{-(a+b)x}) r_k(y) dxdy + \int_0^\infty \int_0^\infty e^{-\varepsilon x - \eta y}\, c_3 (\, e^{-(a-b)x} - e^{-(a+b)x}) \sum_{i=0}^{k-1} \int_0^x g_i(u) \bar{G}(x-u) du\, r_i(y) dxdy$$

We get

$$f_{T,R}^*(\varepsilon, \eta) = c_1\, g^{*k}(\varepsilon + a - b) r^{*k}(\eta) + c_2\, g^{*k}(\varepsilon + a + b) r^{*k}(\eta) + c_3 \sum_{i=0}^{k-1} g^{*i}(\varepsilon + a - b)\, \overline{G^*}(\varepsilon + a - b) r^{*i}(\eta) - c_3 \sum_{i=0}^{k-1} g^{*i}(\varepsilon + a + b)\, \overline{G^*}(\varepsilon + a + b) r^{*i}(\eta)$$

$$f_{T,R}^*(\varepsilon, \eta) = c_1\, g^{*k}(\varepsilon + a - b) r^{*k}(\eta) + c_2\, g^{*k}(\varepsilon + a + b) r^{*k}(\eta) + c_3 \left[\frac{1-(g^*(\varepsilon+a-b) r^*(\eta))^k}{1-g^*(\varepsilon+a-b) r^*(\eta)}\right] [\overline{G}(\varepsilon + a - b)] - c_3 \left[\frac{1-(g^*(\varepsilon+a+b) r^*(\eta))^k}{1-g^*(\varepsilon+a+b) r^*(\eta)}\right] [\overline{G^*}(\varepsilon + a + b)] \qquad (7)$$

The Laplace transform of T is

$$f_{T,R}^*(\varepsilon, 0) = c_1 g^{*k}(\varepsilon + a - b) + c_2\, g^{*k}(\varepsilon + a + b) + c_3 \left[\frac{1-(g^*(\varepsilon+a-b))^k}{1-g^*(\varepsilon+a-b)}\right] [\overline{G^*}(\varepsilon + a - b)] - c_3 \left[\frac{1-(g^*(\varepsilon+a+b))^k}{1-g^*(\varepsilon+a+b)}\right] [\overline{G^*}(\varepsilon + a + b)] \qquad (8)$$

On differentiation of equation (7) we get,

$\frac{\partial}{\partial \varepsilon} f_{T,R}^*(0,0) = -E(T)$ and we obtain

$$
\begin{aligned}
E(T) = &-c_1\, \mathrm{g}^{*\mathrm{k}-1}(a-b)\mathrm{g}^{*\prime}(a-b) - c_2\, \mathrm{g}^{*\mathrm{k}-1}(a+b)\mathrm{g}^{*\prime}(a+b) \\
&+ c_3\left[\frac{\mathrm{k}\, \mathrm{g}^{*\mathrm{k}-1}(a-b)\mathrm{g}^{*\prime}(a-b)}{1-\mathrm{g}^{*}(a-b)}\right][\overline{\mathrm{G}^{*}}(a-b)] \\
&- c_3\left[\frac{1-\mathrm{g}^{*\mathrm{k}}(a-b)}{\left(1-\mathrm{g}^{*}(a-b)\right)^2}\right] \mathrm{g}^{*\prime}(a-b)\, [\overline{\mathrm{G}^{*}}(a-b)] \\
&- c_3\left[\frac{1-\mathrm{g}^{*\mathrm{k}}(a-b)}{\left(1-\mathrm{g}^{*}(a-b)\right)}\right]\left[\overline{\mathrm{G}^{*}}'(a-b)\right] \\
&- c_3\left[\frac{\mathrm{k}\, \mathrm{g}^{*\mathrm{k}-1}(a+b)\mathrm{g}^{*\prime}(a+b)}{1-\mathrm{g}^{*}(a-b)}\right][\overline{\mathrm{G}^{*}}(a+b)] \\
&+ c_3\left[\frac{1-\mathrm{g}^{*\mathrm{k}}(a+b)}{\left(1-\mathrm{g}^{*}(a+b)\right)^2}\right] \mathrm{g}^{*\prime}(a+b)\, [\overline{\mathrm{G}^{*}}(a+b)] \\
&+ c_3\left[\frac{1-\mathrm{g}^{*\mathrm{k}}(a+b)}{\left(1-\mathrm{g}^{*}(a+b)\right)}\right]\left[\overline{\mathrm{G}^{*}}'(a+b)\right]
\end{aligned} \tag{9}
$$

Similarly we note $\frac{\partial}{\partial\eta} f^{*}_{T,R}(0,0) = -\mathrm{E(R)}$ and we obtain

$$
\begin{aligned}
\mathrm{E(R)} = &\; c_1\, \mathrm{g}^{*\mathrm{k}}(a-b)\mathrm{kE(R_1)} + c_2\, \mathrm{g}^{*\mathrm{k}}(a+b)\mathrm{kE(R_1)} \\
&- c_3\mathrm{kE(R_1)}\left[\frac{\mathrm{g}^{*\mathrm{k}}(a-b)}{1-\mathrm{g}^{*}(a-b)}\right][\overline{\mathrm{G}^{*}}(a-b)] \\
&+ c_3\mathrm{E(R_1)}\left[\frac{1-\mathrm{g}^{*\mathrm{k}}(a-b)}{\left(1-\mathrm{g}^{*}(a-b)\right)^2}\right]\mathrm{g}^{*}(a-b)\, [\overline{\mathrm{G}^{*}}(a-b)] \\
&+ c_3\mathrm{kE(R_1)}\left[\frac{\mathrm{g}^{*\mathrm{k}}(a+b)}{1-\mathrm{g}^{*}(a+b)}\right][\overline{\mathrm{G}^{*}}(a+b)] \\
&- c_3\mathrm{E(R_1)}\left[\frac{1-\mathrm{g}^{*\mathrm{k}}(a+b)}{\left(1-\mathrm{g}^{*}(a+b)\right)^2}\right]\mathrm{g}^{*}(a+b)\, [\overline{\mathrm{G}^{*}}(a+b)]
\end{aligned} \tag{10}
$$

We now consider the special case in which the distribution of the production time of a product G(x) is exponential with parameter 'α'. This gives

$$
g^{*}(a-b) = \frac{\alpha}{\alpha+a-b} \qquad g^{*\prime}(a-b) = \frac{-\alpha}{(\alpha+a-b)^2}
$$

$$
\overline{\mathrm{G}^{*}}(a-b) = \frac{1}{\alpha+a-b} \qquad \overline{\mathrm{G}^{*}}'(a-b) = \frac{-1}{(\alpha+a-b)^2} \tag{11}
$$

Using (9), (10) & (11) we find E (T) & E(R),

$$
E(T) = k\left(\frac{\alpha}{(\alpha+a-b)}\right)^k \left(\frac{1}{(\alpha+a-b)}\right)\left(\frac{-\lambda 1}{2b}\right) + \left[1-\left(\frac{\alpha}{(\alpha+a-b)}\right)^k\right]\left(\frac{1}{(a-b)2b}\right) + k\left(\frac{\alpha}{(\alpha+a+b)}\right)^k \left(\frac{1}{(\alpha+a+b)}\right)\left(\frac{\lambda 1}{2b}\right) - \left[1-\left(\frac{\alpha}{(\alpha+a+b)}\right)^k\right]\left(\frac{1}{(a+b)2b}\right) \tag{12}
$$

and

$$E(R) = kE(R_1)\left(\frac{\alpha}{(\alpha+a-b)}\right)^k \left(\frac{-\lambda 1}{2b}\right) + E(R_1)\left(\frac{\alpha(a+b)}{(a-b)2b}\right)\left[1-\left(\frac{\alpha}{(\alpha+a-b)}\right)^k\right] + kE(R_1)\left(\frac{\alpha}{(\alpha+a+b)}\right)^k\left(\frac{\lambda 1}{2b}\right) - E(R_1)\left[1-\left(\frac{\alpha}{(\alpha+a+b)}\right)^k\right]\left(\frac{\alpha a-b}{(a+b)2b}\right) \quad (13)$$

3. Numerical Example

To illustrate the applications of the above result fixing a=13.5,b=10.5, α=0.2 and varying k.Expected time to sales E(T),Expected sales time E(R) are obtained in the following table. For $E(R_1) = 20$

3.1 2D graphs of E(T) and E(R)

Using formulae (12) & (13) and taking the values a=13.5,b=10.5, α=0.2 and k=1 to 6, E(T) is found and α=5, E(R) is found and the corresponding graphs are drawn.

a	b	α	k	E(T)
13.5	10.5	0.2	1	0.005603
13.5	10.5	0.2	2	0.012899
13.5	10.5	0.2	3	0.013798
13.5	10.5	0.2	4	0.013881
13.5	10.5	0.2	5	0.013888
13.5	10.5	0.2	6	0.013889

Table 3.1: E(T) values

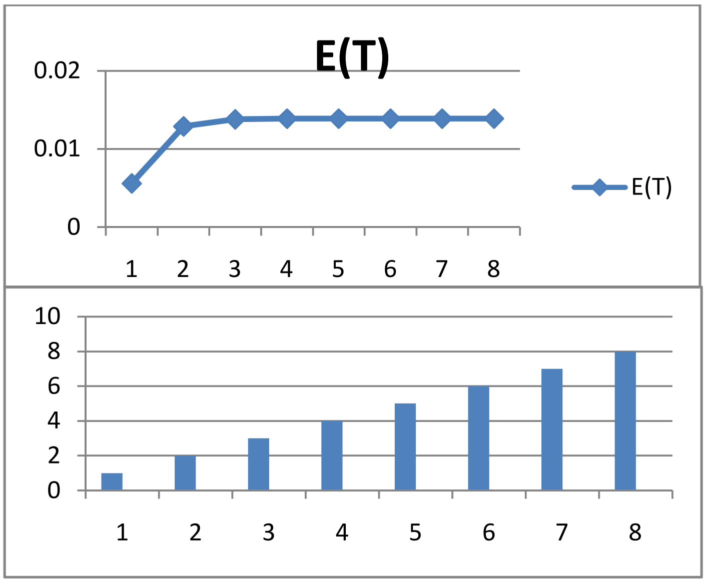

Figure 3.1: 2D graphs of E(T) for table 3.1

a	b	α	k	E (R)
13.5	10.5	5	1	10.34483
13.5	10.5	5	2	17.13734
13.5	10.5	5	3	22.73925
13.5	10.5	5	4	27.06429
13.5	10.5	5	5	30.2398
13.5	10.5	5	6	32.50578

Table 3.2: E(R) values

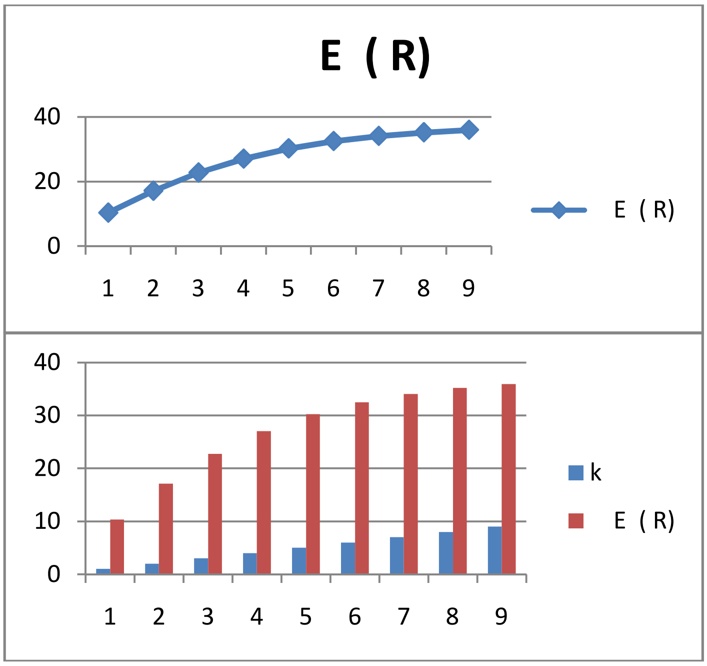

Figure 3.2: 2D graphs of E(R) for table 3.2

Observation

As k increase from 1 to 6, Expected time to sales E(T) increases, Expected sales time E(R) also increases.

3.3 3D graphs of E(T) and E(R)

Taking a=13.5, b=10.5, α=0.2 to 0.7 & α=5,6,….10 and k=1 to 6, 3D graphs of E(T) and E(R) are drawn.

	α=.2, E(T)	α=.3, E(T)	α=.4, E(T)	α=.5, E(T)	α=.6, E(T)	α=.7, E(T)
k						
1	0.005603	0.002169	-0.00087	-0.00357	-0.00597	-0.0081
2	0.012899	0.011855	0.010577	0.009136	0.007589	0.005977
3	0.013798	0.013617	0.013316	0.012891	0.012346	0.011691
4	0.013881	0.013856	0.0138	0.013701	0.01355	0.013341
5	0.013888	0.013885	0.013876	0.013856	0.013819	0.01376
6	0.013889	0.013888	0.013887	0.013883	0.013875	0.01386

Table 3.3a: E(T) values

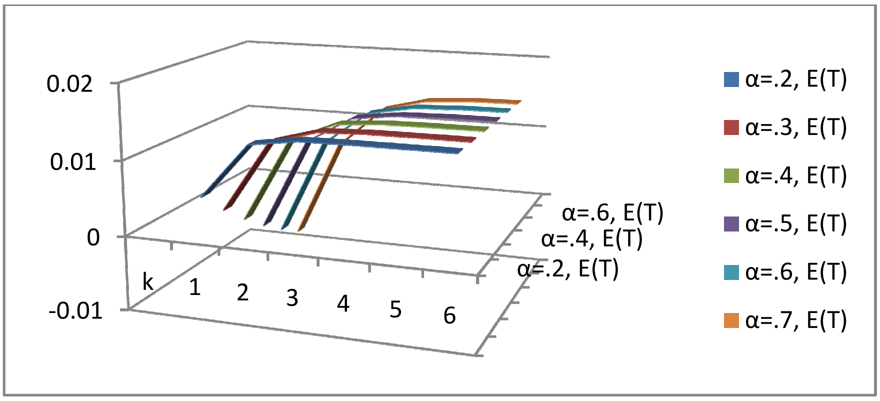

Figure 3.3a: 3D graphs of E(T) for table 3.3

	α=5, E (R)	α=6, E (R)	α=7, E (R)	α=8, E (R)	α=9, E (R)	α=10, E (R)
k						
1	10.34483	11.11111	11.74194	12.2727	12.72727	13.12217
2	17.13734	18.54815	19.71946	20.7128	21.57025	22.32141
3	22.73925	24.87111	26.63943	28.1327	29.41397	30.52871
4	27.06429	29.99993	32.45873	34.5445	36.33525	37.89
5	30.2398	33.97577	37.15645	39.8855	42.2466	44.306
6	32.50578	36.97632	40.85332	44.2275	47.17885	49.7746

Table 3.3b: E(R) values

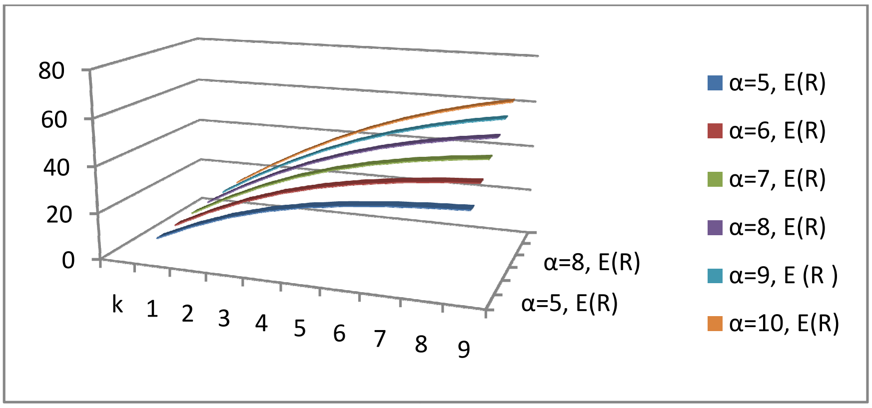

Figure 3.3b: 3D graphs of E(R) for table 3.3

Observation

When k increases and α is fixed, the expected time to sales E(T) increases and sales time E(R) increases.

When k is fixed and α increases, the expected time to sales E(T) decreases and sales time E(R) increases.

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