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AN IMPROVED RATIO-CUM-PRODUCT ESTIMATOR OF POPULATION MEAN USING COEFFICIENT OF KURTOSIS OF THE AUXILIARY VARIATES IN STRATIFIED RANDOM SAMPLING

Rajesh Tailor*, Arpita Lakhre*, Ritesh Tailor[1] and Neha Garg[2]
*S. S. in Statistics, Vikram University, Ujjain, India
[1]Policy Research and Marketing Division, Institute of Wood Science and Technology, Bangalore, India
[2]School of Sciences, Indira Gandhi National Open University (IGNOU), New Delhi, India
E Mail: tailorraj@gmail.com, arpita.1.lakhre@gmail.com, riteshntailor@gmail.com, nehagarg@ignou.ac.in

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Abstract

In this paper, we have discussed the problem of estimation of population mean in stratified random sampling. An Improved ratio-cum-product estimator of population mean by using information on known coefficient of kurtosis of auxiliary variate has been suggested. The suggested estimator has been compared with usual unbiased estimator, combined ratio and product estimators, Kadilar and Cingi (2003) ratio and product type estimators and Tailor et al. (2012) ratio-cum-product estimator. The conditions under which the suggested estimator is more efficient have been obtained. An empirical study has been carried out to demonstrate the performance of the suggested estimator.

Key Words: Finite Population Mean, Stratified Random Sampling, Auxiliary Variate, Bias, Mean Squared Error.

1. Introduction

Hansen et al. (1946) defined combined ratio estimator for estimating the population mean. Kadilar and Cingi (2003) defined a ratio-type estimator for population mean using coefficient of variation and coefficient of kurtosis of auxiliary variate. Singh et al. (2008) suggested a class of estimators of population mean using power transformation in stratified random sampling. Singh (1967) utilized information on population mean of two auxiliary variates and envisaged ratio-cum-product estimator for population mean. Tailor et al. (2012) studied Singh (1967) estimator in stratified random sampling. Tailor et al. (2013) discussed dual to ratio and product type exponential estimators of population mean in stratified random sampling. Parmar (2013) studied a ratio-cum-product estimator of population mean in stratified random sampling using coefficient of variation of auxiliary variates. Parmar (2013) motivated us to study an alternative estimator of population mean using coefficient of kurtosis of auxiliary variates in stratified random sampling.

Let us consider a finite population $\mathrm{U}=\{\mathrm{U}_1,\mathrm{U}_2,...,\mathrm{U}_\mathrm{N}\}$ of size N. This population is divided into L homogenous strata of sizes $N_h(h=1,2,...,L)$. Let y be the study variate and x and z are auxiliary variates taking values y_{hi}, x_{hi} and $z_{hi}(h=1,2,...,L;\ i=1,2,...,N_h)$ on i^{th} unit of the h^{th} stratum. A sample of size n_h is drawn from each stratum which constitutes a sample of size $n=\sum_{h=1}^{L}n_h$. Then we define

$\overline{Y}_h=\frac{1}{N_h}\sum_{i=1}^{N_h}y_{hi}$: h^{th} Population mean of the study variate y in h^{th} stratum,

$\overline{X}_h=\frac{1}{N_h}\sum_{i=1}^{N_h}x_{hi}$: h^{th} Population mean of the auxiliary variate x in h^{th} stratum,

$\overline{Z}_h=\frac{1}{N_h}\sum_{i=1}^{N_h}z_{hi}$: h^{th} stratum mean for the auxiliary variate z,

$\overline{Y}=\frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h}y_{hi}=\frac{1}{N}\sum_{h=1}^{L}N_h\overline{Y}_h=\sum_{h=1}^{L}W_h\overline{Y}_h$: Population mean of the study variate y,

$\overline{X}=\frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h}x_{hi}=\frac{1}{N}\sum_{h=1}^{L}W_h\overline{X}_h$: Population mean of the auxiliary variate x,

$\overline{Z}=\frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h}z_{hi}=\frac{1}{N}\sum_{h=1}^{L}W_h\overline{Z}_h$: Population mean of the auxiliary variate z,

$\overline{y}_h=\frac{1}{n_h}\sum_{i=1}^{n_h}y_{hi}$: Sample mean of the study variate y for h^{th} stratum,

$\overline{x}_h=\frac{1}{n_h}\sum_{i=1}^{n_h}x_{hi}$: Sample mean of the auxiliary variate x for h^{th} stratum,

$\overline{z}_h=\frac{1}{n_h}\sum_{i=1}^{n_h}z_{hi}$: Sample mean of the auxiliary variate z for h^{th} stratum,

$W_h=\frac{N_h}{N}$: Stratum weight of h^{th} stratum,

Cochran (1940) envisaged ratio estimator for estimating the population mean of the study variate y as

$$\hat{\overline{\mathrm{Y}}}_{\mathrm{R}}=\overline{\mathrm{y}}\left(\frac{\overline{\mathrm{X}}}{\overline{\mathrm{x}}}\right) \tag{1.1}$$

In the line of Cochran (1940), in stratified random sampling, Hansen et.al (1946) envisaged combined ratio estimator for estimating the population mean $\overline{Y}$ as

$$\hat{\overline{Y}}_{RC} = \overline{y}_{st}\left(\frac{\overline{X}}{\overline{x}_{st}}\right). \tag{1.2}$$

In case of negative correlation coefficient between the study variate y and auxiliary variate z, combined product estimator is defined as

$$\hat{\overline{Y}}_{PC} = \overline{y}_{st}\left(\frac{\overline{z}_{st}}{\overline{Z}}\right), \tag{1.3}$$

where $\overline{y}_{st} = \sum_{h=1}^{L} W_h \overline{y}_h$, $\overline{x}_{st} = \sum_{h=1}^{L} W_h \overline{x}_h$ and $\overline{z}_{st} = \sum_{h=1}^{L} W_h \overline{z}_h$ are unbiased estimators of population mean $\overline{Y}$, $\overline{X}$ and $\overline{Z}$ respectively.

Using the information on population mean of two auxiliary variates, Singh (1967) suggested a ratio-cum-product estimator for population mean $\overline{Y}$ as

$$\hat{\overline{Y}}_{RP} = \overline{y}\left(\frac{\overline{X}}{\overline{x}}\right)\left(\frac{\overline{z}}{\overline{Z}}\right) \tag{1.4}$$

where $\overline{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$, $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ and $\overline{z} = \frac{1}{n}\sum_{i=1}^{n} z_i$ are unbiased estimators of population means $\overline{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i$, $\overline{X} = \frac{1}{N}\sum_{i=1}^{N} x_i$ and $\overline{Z} = \frac{1}{N}\sum_{i=1}^{N} z_i$ respectively.

Singh et al. (2004) suggested modified ratio and product estimators, using coefficient of kurtosis of auxiliary variates as

$$\hat{\overline{Y}}_{STR} = \overline{y}\left(\frac{\overline{X} + \beta_2(x)}{\overline{x} + \beta_2(x)}\right) \tag{1.5}$$

and

$$\hat{\overline{Y}}_{STP} = \overline{y}\left(\frac{\overline{Z} + \beta_2(z)}{\overline{z} + \beta_2(z)}\right) \tag{1.6}$$

where $\beta_2(x)$ and $\beta_2(z)$ are coefficient of kurtosis of the auxiliary variates x and z respectively.

Sharma (2012) defined a ratio-cum-product estimator of population mean using coefficient of kurtosis of two auxiliary variates x and z i.e. $\beta_2(x)$ and $\beta_2(z)$ respectively as

$$\hat{\overline{Y}}_{RPI} = \overline{y}\left(\frac{\overline{X} + \beta_2(x)}{\overline{x} + \beta_2(x)}\right)\left(\frac{\overline{z} + \beta_2(z)}{\overline{Z} + \beta_2(z)}\right) \tag{1.7}$$

Singh et al. (2008) defined a combined ratio type estimator $\hat{\bar{Y}}_{SE}$ in stratified random sampling using coefficient of kurtosis in each stratum as

$$\hat{\bar{Y}}_{SER}^{ST} = \bar{y}_{st}\left(\frac{\sum_{h=1}^{L} W_h\left(\bar{X}_h + \beta_{2h}(x)\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_h + \beta_{2h}(x)\right)}\right) \quad (1.8)$$

Combined product type estimator using coefficient of kurtosis is defined as

$$\hat{\bar{Y}}_{SEP}^{ST} = \bar{y}_{st}\left(\frac{\sum_{h=1}^{L} W_h\left(\bar{z}_h + \beta_{2h}(z)\right)}{\sum_{h=1}^{L} W_h\left(\bar{Z}_h + \beta_{2h}(z)\right)}\right) \quad (1.9)$$

Tailor et al. (2012) defined Singh (1967) estimator $\hat{\bar{Y}}_{RP}$ in stratified random sampling as

$$\hat{\bar{Y}}_{RP}^{ST} = \left(\frac{\bar{X}}{\bar{x}_{st}}\right)\left(\frac{\bar{z}_{st}}{\bar{Z}}\right) \quad (1.10)$$

In this paper the work of Tailor et al. (2012) has been extended and a ratio-cum-product estimator has been suggested in stratified random sampling using information on coefficient of kurtosis in each stratum.

2. Suggested Estimator

An improved ratio-cum-product estimator of population mean $\bar{Y}$ using coefficient of kurtosis of auxiliary variates, in each stratum is suggested as

$$\hat{\bar{Y}}_{RP1}^{ST} = y_{st}\left(\frac{\sum_{h=1}^{L} W_h\left(\bar{X}_h + \beta_{2h}(x)\right)}{\sum_{h=1}^{L} W_h\left(\bar{x}_h + \beta_{2h}(x)\right)}\right)\left(\frac{\sum_{h=1}^{L} W_h\left(\bar{z}_h + \beta_{2h}(z)\right)}{\sum_{h=1}^{L} W_h\left(\bar{Z}_h + \beta_{2h}(z)\right)}\right) \quad (2.1)$$

where $\beta_{2h}(x)$ and $\beta_{2h}(z)$ are coefficients of kurtosis of auxiliary variates x and z in h^{th} stratum respectively.

To obtain the bias and mean squared error of the suggested estimator $\hat{\bar{Y}}_{RP1}^{ST}$, we write $\bar{y}_h = \bar{Y}_h(1+e_{0h})$, $\bar{x}_h = \bar{X}_h(1+e_{1h})$ and $\bar{z}_h = \bar{Z}_h(1+e_{2h})$ such that

$$\mathrm{E}(e_{0h}) = \mathrm{E}(e_{1h}) = \mathrm{E}(e_{2h}) = 0$$

$$\mathrm{E}(e_{0h}^2) = \gamma_h C_{yh}^2,\ \mathrm{E}(e_{1h}^2) = \gamma_h C_{xh}^2,\ \mathrm{E}(e_{2h}^2) = \gamma_h C_{zh}^2$$

$$\mathrm{E}(e_{0h}e_{1h}) = \gamma_h \rho_{yxh} C_{yh} C_{xh} = \frac{\gamma_h S_{yxh}}{\bar{X}_h \bar{Y}_h},\ \mathrm{E}(e_{0h}e_{2h}) = \gamma_h \rho_{yzh} C_{yh} C_{zh} = \frac{\gamma_h S_{yzh}}{\bar{Y}_h \bar{Z}_h}$$

$$E(e_{1h}e_{2h}) = \gamma_h \rho_{xzh} C_{xh} C_{zh} = \frac{\gamma_h S_{xzh}}{\overline{X}_h \overline{Z}_h} \text{ and } \gamma_h = \left(\frac{1}{n_h} - \frac{1}{N_h}\right).$$

Now, suggested estimator $\hat{\overline{Y}}_{RP1}^{ST}$ can be expressed as

$$\left(\hat{\overline{Y}}_{RP1}^{ST} - \overline{Y}\right) = \overline{Y}\left(e_0 - e_1 + e_2 + e_1^2 - e_1 e_2 - e_0 e_1 + e_0 e_2\right) \tag{2.2}$$

where

$$e_0 = \frac{\sum_{h=1}^{L} W_h \overline{Y}_h e_{0h}}{\overline{Y}}, \quad e_1 = \frac{\sum_{h=1}^{L} W_h \overline{X}_h e_{1h}}{X_{SE}}, \quad e_2 = \frac{\sum_{h=1}^{L} W_h \overline{Y}_h e_{2h}}{Z_{SE}},$$

$$E(e_0^2) = \frac{1}{\overline{Y}^2} \sum_{h=1}^{L} W_h^2 \gamma_h S_{yh}^2, \quad E(e_1^2) = \frac{1}{X_{SE}^2} \sum_{h=1}^{L} W_h^2 \gamma_h S_{xh}^2,$$

$$E(e_2^2) = \frac{1}{Z_{SE}^2} \sum_{h=1}^{L} W_h^2 \gamma_h S_{zh}^2, \quad E(e_0 e_1) = \frac{1}{\overline{Y} X_{SE}} \sum_{h=1}^{L} W_h^2 \gamma_h S_{yxh},$$

$$E(e_0 e_2) = \frac{1}{\overline{Y} Z_{SE}} \sum_{h=1}^{L} W_h^2 \gamma_h S_{yzh}, \quad E(e_1 e_2) = \frac{1}{X_{SE} Z_{SE}} \sum_{h=1}^{L} W_h^2 \gamma_h S_{xzh},$$

$$X_{SE} = \sum_{h=i}^{L} W_h \left(\overline{X}_h + \beta_{2h}(x)\right), \text{ and } Z_{SE} = \sum_{h=i}^{L} W_h \left(\overline{Z}_h + \beta_{zh}(z)\right).$$

Using standard procedure, the bias and mean squared error of suggested estimator, up to the first degree of approximation are obtained, as

$$B\left(\hat{\overline{Y}}_{RP1}^{ST}\right) = \overline{Y} \sum_{h=1}^{L} W_h^2 \gamma_h \left(\frac{S_{xh}^2}{X_{SE}^2} - \frac{S_{yxh}}{\overline{Y} X_{SE}} - \frac{S_{xzh}}{X_{SE} Z_{SE}} + \frac{S_{yzh}}{\overline{Y} Z_{SE}}\right) \tag{2.3}$$

$$MSE\left(\hat{\overline{Y}}_{RP1}^{ST}\right) = \overline{Y}^2 \sum_{h=1}^{L} W_h^2 \gamma_h \left(\frac{S_{yh}^2}{\overline{Y}^2} + \frac{S_{xh}^2}{X_{SE}^2} + \frac{S_{zh}^2}{Z_{SE}^2} - \frac{2S_{yxh}}{\overline{Y} X_{SE}} + \frac{2S_{yzh}}{\overline{Y} Z_{SE}} + \frac{2S_{xzh}}{X_{SE} Z_{SE}}\right) \tag{2.4}$$

which can also be written as

$$MSE\left(\hat{\overline{Y}}_{RP1}^{ST}\right) = \sum_{h=1}^{L} W_h^2 \gamma_h \left(S_{yh}^2 + R_{1SE}^2 S_{xh}^2 + R_{2SE}^2 S_{zh}^2 - 2R_{1SE} S_{yxh} + 2R_{2SE} S_{yzh} - 2R_{1SE} R_{2SE} S_{xzh}\right) \tag{2.5}$$

where

$$R_{1SE} = \frac{\overline{Y}}{X_{SE}} \text{ and } R_{2SE} = \frac{\overline{Y}}{Z_{SE}}.$$

3. Efficiency Comparisons

The variance of usual unbiased estimator $\overline{y}_{st}$ of population mean $\overline{Y}$ in stratified random sampling is defined as

$$V\left(\bar{y}_{st}\right)=\sum_{h=1}^{L} W_h^2 \gamma_h S_{yh}^2 \tag{3.1}$$

Mean squared errors of $\hat{\bar{Y}}_{RC}$, $\hat{\bar{Y}}_{PC}$, $\hat{\bar{Y}}_{SER}^{ST}$, $\hat{\bar{Y}}_{SEP}^{ST}$ and $\hat{\bar{Y}}_{RP}^{ST}$ are expressed as

$$MSE\left(\hat{\bar{Y}}_{RC}\right)=\sum_{h=1}^{L} W_h^2 \gamma_h \left(S_{yh}^2+R_1^2 S_{xh}^2-2R_1 S_{yxh}\right), \tag{3.2}$$

$$MSE\left(\hat{\bar{Y}}_{PC}\right)=\sum_{h=1}^{L} W_h^2 \gamma_h \left(S_{yh}^2+R_2^2 S_{zh}^2+2R_2 S_{yzh}\right), \tag{3.3}$$

$$\mathrm{MSE}\left(\hat{\bar{\mathrm{Y}}}_{\mathrm{SER}}^{\mathrm{ST}}\right)=\sum_{\mathrm{h}=1}^{\mathrm{L}} \mathrm{W}_{\mathrm{h}}^2 \gamma_{\mathrm{h}} \left(\mathrm{S}_{\mathrm{yh}}^2+\mathrm{R}_{1\mathrm{SE}}^2 \mathrm{S}_{\mathrm{xh}}^2-2\mathrm{R}_{1\mathrm{SE}} \mathrm{S}_{\mathrm{yxh}}\right), \tag{3.4}$$

$$\mathrm{MSE}\left(\hat{\bar{\mathrm{Y}}}_{\mathrm{SEP}}^{\mathrm{ST}}\right)=\sum_{\mathrm{h}=1}^{\mathrm{L}} \mathrm{W}_{\mathrm{h}}^2 \gamma_{\mathrm{h}} \left(\mathrm{S}_{\mathrm{yh}}^2+\mathrm{R}_{2\mathrm{SE}}^2 \mathrm{S}_{\mathrm{zh}}^2+2\mathrm{R}_{2\mathrm{SE}} \mathrm{S}_{\mathrm{yzh}}\right), \tag{3.5}$$

$$MSE\left(\hat{\bar{Y}}_{RP}^{ST}\right)=\sum_{h=1}^{L} W_h^2 \gamma_h \left(S_{yh}^2+R_1^2 S_{xh}^2+R_2^2 S_{zh}^2+2\left(R_1 S_{yxh}-R_2 S_{yzh}+R_1 R_2 S_{xzh}\right)\right) \tag{3.6}$$

where

$$R_1=\frac{\bar{Y}}{\bar{X}} \text{ and } R_2=\frac{\bar{Y}}{\bar{Z}}$$

$$\mathrm{X}_{\mathrm{SE}}=\sum_{\mathrm{h}=1}^{\mathrm{L}} \mathrm{W}_{\mathrm{h}}\left(\bar{\mathrm{X}}_{\mathrm{h}}+\beta_{2\mathrm{h}}(\mathrm{x})\right) \text{ and } \mathrm{Z}_{\mathrm{SE}}=\sum_{\mathrm{h}=1}^{\mathrm{L}} \mathrm{W}_{\mathrm{h}}\left(\bar{\mathrm{Z}}_{\mathrm{h}}+\beta_{\mathrm{zh}}(\mathrm{z})\right).$$

Comparison of (2.5), (3.1), (3.2), (3.3), (3.4), (3.5) and (3.6) shows that the suggested estimator $\hat{\bar{Y}}_{RP1}^{ST}$ would be more efficient than

(i) usual unbiased estimator $\bar{y}_{st}$ i.e.

$$MSE\left(\hat{\bar{Y}}_{RP1}^{ST}\right)-V\left(\bar{y}_{st}\right)<0 \text{ if}$$

$$\mathrm{R}_{1\mathrm{SE}}\mathrm{A}\left(\mathrm{R}_{1\mathrm{SE}}-2\mathrm{C}\right)+\mathrm{R}_{2\mathrm{SE}}\left(\mathrm{R}_{2\mathrm{SE}}\mathrm{B}+\mathrm{D}\right)-2\mathrm{R}_{1\mathrm{SE}}\mathrm{R}_{2\mathrm{SE}}\mathrm{E}<0, \tag{3.7}$$

(ii) combined ratio estimator $\hat{\bar{Y}}_{RC}$ i.e.

$$\mathrm{MSE}\left(\hat{\bar{\mathrm{Y}}}_{\mathrm{RP1}}^{\mathrm{ST}}\right)-\mathrm{MSE}\left(\hat{\bar{\mathrm{Y}}}_{\mathrm{RC}}\right)<0 \text{ if}$$

$$\mathrm{A}\left(\mathrm{R}_{1\mathrm{SE}}^2-\mathrm{R}_1^2\right)-2\left(\mathrm{R}_{1\mathrm{SE}}-\mathrm{R}_1\right)\mathrm{C}+\mathrm{R}_{2\mathrm{SE}}^2\mathrm{B}+2\mathrm{R}_{2\mathrm{SE}}^2\mathrm{D}-2\mathrm{R}_{1\mathrm{SE}}\mathrm{R}_{2\mathrm{SE}}\mathrm{E}<0 \tag{3.8}$$

(iii) combined product estimator $\hat{\bar{Y}}_{PC}$ i.e.

$$MSE\left(\hat{\bar{Y}}_{RP1}^{ST}\right)-MSE\left(\hat{\bar{Y}}_{PC}\right)<0 \text{ if}$$

$$AR_{1SE}^{2} + B\left(R_{2SE}^{2} - R_{2}^{2}\right) - 2R_{1SE}C + 2D\left(R_{2SE} - R_{2}\right) - 2R_{1SE}R_{2SE}E < 0 \tag{3.9}$$

(iv) Kadilar and Cingi (2003) ratio type estimator $\hat{\bar{Y}}_{SER}^{ST}$ i.e.

$$MSE\left(\hat{\bar{Y}}_{RP1}^{ST}\right) - MSE\left(\hat{\bar{Y}}_{SER}^{ST}\right) < 0 \text{ if}$$
$$R_{2SE}B + 2D - 2R_{1SE}E < 0, \tag{3.10}$$

(v) Kadilar and Cingi (2003) product type estimator of $\hat{\bar{Y}}_{SEP}^{ST}$ i.e.

$$MSE\left(\hat{\bar{Y}}_{RP1}^{ST}\right) - MSE\left(\hat{\bar{Y}}_{SEP}^{ST}\right) < 0 \text{ if}$$
$$R_{1SE}A - 2C - 2R_{2SE}E < 0 \tag{3.11}$$

(vi) Tailor et al. (2012) estimator $\hat{\bar{Y}}_{RP}^{ST}$ i.e.

$$MSE\left(\hat{\bar{Y}}_{RP1}^{ST}\right) - MSE\left(\hat{\bar{Y}}_{RP}^{ST}\right) < 0 \text{ if}$$
$$\left(R_{1SE} - R_{1}\right)\left\{A\left(R_{1SE} - R_{1}\right) - 2C\right\} + \left(R_{2SE} - R_{2}\right)\left\{B\left(R_{2SE} + R_{2}\right) - 2C\right\}$$
$$-2E\left(R_{1SE}R_{2SE} - R_{1}R_{2}\right) < 0 \tag{3.12}$$

Expressions (3.7) to (3.12) are conditions under which the suggested ratio-cum-product estimator would be more efficient than $\hat{\bar{Y}}_{RC}$, $\hat{\bar{Y}}_{PC}$, $\hat{\bar{Y}}_{SER}^{ST}$, $\hat{\bar{Y}}_{SEP}^{ST}$ and $\hat{\bar{Y}}_{RP}^{ST}$ respectively.

4. Empirical Study

To see the performance of the suggested estimator $\hat{\bar{Y}}_{RP1}^{ST}$, two natural population data sets are being considered. Description of the populations are given below:

Population I[Source: Murthy (1967)]

y : Output, x : Fixed capital and z : Number of workers.

N=10 n=4	n_1=2	n_2=2	N_1=5	N_2=5
	$\bar{Z}_1$=51.80	$\bar{Z}_2$=60.60	$\bar{X}_1$=214.40	$\bar{X}_2$=33 80
	$\bar{Y}_1$= 1325.80	$\bar{Y}_2$=315.60	S_{z_1}=0.75	S_{z_2}=4.84
	S_{x_1}=74.87	S_{x_2}=66.35	S_{y_1}=615.32	S_{y_2}=340.38
	S_{zx_1}=38.08	S_{zx_2}=287.32	S_{yz_1}=411.16	S_{yz_2}=1536.24
	S_{yx_1}=33360.68	S_{yx_2}=22356.52		

Population II [Source: National Horticulture Board]
y : Productivity (MT/ Hectare) , x : Production in '000 Tons and
z : Area in '000 Hectare

N=20 n=8	n_1=4	n_2=4	N_1=10	N_2=10
	$\overline{Z}_1$=6.20	$\overline{Z}_2$=80.67	$\overline{X}_1$=10.41	$\overline{X}_2$=30 14
	$\overline{Y}_1$= 1.70	$\overline{Y}_2$= 67	S_{z_1}=1.13	S_{z_2}=10.81
	S_{x_1}= 53	S_{x_2}=80.54	S_{y_1}=0.54	S_{y_2}=1.41
	S_{zx_1}=1.75	S_{zx_2}=68.57	S_{yz_1}=-0.02	S_{yz_2}=-7.06
	S_{yx_1}=1.60	S_{yx_2}=8 47		

Estimator	$\overline{y}_{st}$	$\hat{\overline{Y}}_{RC}$	$\hat{\overline{Y}}_{PC}$	$\hat{\mathrm{Y}}_{\mathrm{SER}}^{\mathrm{ST}}$	$\hat{\mathrm{Y}}_{\mathrm{SEP}}^{\mathrm{ST}}$	$\hat{\overline{Y}}_{RP}^{ST}$	$\hat{\overline{Y}}_{RP1}^{ST}$
Population I	100.00	239.88	68.90	240.53	20.01	308.58	333.14
Population II	100.00	184.86	123.06	185.12	32.50	343.16	407.27

Table 4.1: Percent relative efficiencies of $\overline{y}_{st}$, $\hat{\overline{Y}}_{RC}$, $\hat{\overline{Y}}_{PC}$, $\hat{\mathrm{Y}}_{\mathrm{SER}}^{\mathrm{ST}}$, $\hat{\mathrm{Y}}_{\mathrm{SEP}}^{\mathrm{ST}}$, $\hat{\overline{Y}}_{RP}^{ST}$ and $\hat{\overline{Y}}_{RP1}^{ST}$ with respect of $\overline{y}_{st}$

Section 3 provides the conditions under which suggested ratio-cum-product type estimators of population mean $\hat{\overline{Y}}_{RP1}^{ST}$ has less mean squared error than that of usual unbiased estimator, combined ratio and product estimators, Kadilar and Cingi (2003) estimators $\hat{\mathrm{Y}}_{\mathrm{SER}}^{\mathrm{ST}}$ and $\hat{\mathrm{Y}}_{\mathrm{SEP}}^{\mathrm{ST}}$ and Tailor et al. (2012) estimators $\hat{\mathrm{Y}}_{\mathrm{RP}}^{\mathrm{ST}}$.

Table 4.1 exhibits that the suggested ratio-cum-product type estimator $\hat{\overline{Y}}_{RP1}^{ST}$ has highest percent relative efficiency as compared to other considered estimators. Thus it can be concluded that if information on coefficient of kurtosis of auxiliary variate is available for each stratum and conditions obtained in section 3 are satisfied, suggested estimator may be an alternative for estimation of population mean.

Conclusion

We have suggested an improved ratio-cum-product estimator of population mean by using information on known coefficient of kurtosis of auxiliary variate for estimation of population mean in stratified random sampling. The suggested estimator is more efficient than usual unbiased estimator, combined ratio and product estimators, Kadilar and Cingi (2003) estimators and Tailor et al. (2012) estimators.

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