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GENERAL PRODUCTION AND SALES SYSTEM WITH SCBZ MACHINE TIME AND MARKOVIAN MANPOWER

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Abstract: A study on production and sales system is performed. A machine produces random number of products during the operation time. After the operation time, sale time starts which has one among two distinct distributions depending on the magnitude of production time within or exceeding a random threshold magnitude. Two models are treated considering manpower system exposed to a departure process; the machine has SCBZ failure pattern with repair; manpower recruitments and sales time have general distributions. The system fails when both manpower and machine are in failed state. In model 1, the sales are done one by one and in model 2, when the operation time is more than a threshold, the sales are done altogether and when it is less than the threshold, the sales are done one by one. The Joint Laplace transform of the variables, their means and the Co-variances with numerical results are presented.

Mathematics Subject Classification: 91B70

Key Words: Storage System, Production and Sale, Repair and Recruitment, Joint Laplace Transform.

1. Introduction

In manufacturing models to get the return on investment and to pay minimum interest, it is natural that when the production time is more, the sale time is made short so as to cut cost. It has been noticed that when the units produced are more, financial supports for the customers are provided to clear products early. These are widely felt in perishable commodity sectors where many banking institutions provide required finance for the purchase.

Storage systems of (s, S) type has been studied by Arrow et al. [1]. Such systems with random lead times and unit demand have been treated by Danial and Ramanarayanan [2]. Models with bulk demands have been analyzed by Ramanarayanan and Jacob [12]. Murthy and Ramanarayanan [8, 9, 10, 11] have considered several (s, S) inventory systems. Kun-Shan Wu et al. [3] studied (Q, r, L) inventory model with defective items. Usha et al. [14] have considered storage systems with random sales time depending on production. General Manpower and Machine system with Markovian production has been analyzed by Hari kumar.k [4]. General Production and Sales System with SCBZ Machine Time and manpower has been studied by Madhusoodhanan at al. [5,6]. Snehalatha et al. [13] have analyzed general manpower and general recruitment system. Madhusoodhanan [7] have analyzed general production and sales by Markovian manpower and machine system.

In this paper, two models are treated. Manpower system is exposed to a departure process, the machine has SCBZ failure pattern with repair, recruitments and sales have general distributions. The system fails when both manpower and machine are in failed state. In model 1, the sales are done one by one and in model 2, when the operation time is more than a threshold, the sales are done altogether and when it is less than the threshold, the sales are done one by one. The Joint Laplace transform of the variables, their means and the co-variances with numerical results are presented.

2. Model 1

2.1 Assumptions

1. The inter production time of products are i.i.d. random variables with Cdf F(x) and pdf f(x). The products are produced by a manpower machine system.

2. The sales times of products are i.i.d random variables with Cdf G(w) and pdf g(w) and the products are sold one by one.

3. The manpower system in-charge of production fails with probability ‘p’ when an employee leaves and survives with probability ‘q’ at the departure. The inter-departure times of employees are i.i.d. random variables having exponential distribution with parameter μ.

4. The machine used for production has SCBZ failure pattern. Its life time has exponential distribution with parameter ‘a’ in phase 1. If the machine does not fail in a random exponential time with parameter ‘c’, it goes to phase 2. In phase 2, the machine has exponential life time distribution with parameter ‘b’.

5. The man power machine system fails, when both are in failed state, when either man power or machine alone is in failed state, the failed one is hired till the other one also fails.

6. When the man power machine system fails, sales begin and man power recruitments are done one by one with recruitment time (R) a random variable with Cdf R(y) and pdf r(y). When the machine fails in phase 1 its repair time R_1 has Cdf $R_1(z)$ and pdf $r_1(z)$. When it fails in phase 2 its repair time R_2 has Cdf $R_2(z)$ and pdf $r_2(z)$.

2.2 Analysis

Since the parameter ‘a’ changes to ‘b’ in an exponential time with parameter ‘c’ if the machine does not fail in phase 1, the life time pdf of machine satisfies the following equation,

$$h(x) = ae^{-ax}e^{-cx} + \int_0^x ce^{-cu}e^{-au}be^{-b(x-u)}du \tag{1}$$

The first term of (1) of R.H.S is the pdf part that the machine fails in phase 1 before the change of parameter. The second term is the pdf part that the machine moves to phase 2 at time u, no failure occurs in phase 1 and the machine fails at x in phase 2.

On simplification the pdf of failure time of the machine is

$$h(x) = ae^{-x(a+c)} + \frac{cb}{c+a-b}(e^{-bx} - e^{-(c+a)x}). \qquad (2)$$

Here the first term is phase 1 failure density and the second term is phase 2 failure density.

The survival probability function s(x) of the machine that it does not fail in phase 1 or in phase 2 satisfies the following equation

$$s(x) = e^{-x(a+c)} + \int_0^x e^{-au} c e^{-cu} e^{-b(x-u)} du. \qquad (3)$$

The first term of right side of equation (2) is the probability that the machine survives in phase 1 and second term is the probability that the machine survives in phase 2. On simplification it may be obtained with two probabilities of phase 1 and phase 2 as follows.

$$s(x) = e^{-x(a+c)} + \frac{c}{c+a-b}(e^{-bx} - e^{-(a+c)x}). \qquad (4)$$

Simplifying equations (2) and (4), it can be seen as

$$h(x) = \alpha(c+a)e^{-x(a+c)} + \beta b e^{-bx} \qquad (5)$$

$$and \;\; s(x) = \alpha e^{-x(a+c)} + \beta e^{-bx}. \qquad (6)$$

$$Here \;\; \alpha = \frac{a-b}{c+a-b}, \beta = \frac{c}{c+a-b} \;\; and \;\; \alpha + \beta = 1. \qquad (7)$$

The structure of equations (2) and (4) present the failure and survival functions in phase1 and phase 2 explicitly for repair and maintenance study.

To study the Model- 1 the joint probability density function of the four variables namely $(X, \hat{R}, \tilde{R}, \hat{S})$ where

i) X is the operation time which is the maximum of life times of machine and manpower
ii) $\hat{R}$ is the sum of recruitment times of employees
iii) $\tilde{R}$ is R_1 or R_2 respectively when the machine fails in phase 1 or phase 2
iv) $\hat{S}$ is the total sales time.

The joint pdf may be seen as

$$f(x,y,z,w) = \begin{Bmatrix} \left[ae^{-x(a+c)} r_1(z) + \frac{cb}{c+a-b}(e^{-bx} - e^{-(c+a)x}) r_2(z) \right] \\ \left[\int_0^x \sum_{i=1}^{\infty} \mu e^{-\mu u} \frac{(\mu u)^{i-1}}{(i-1)!} q^{i-1} pr_i(y) du \right] + \\ \left[r_1(z) \int_0^x ae^{-u(a+c)} du + r_2(z) \frac{cb}{c+a-b} \int_0^x (e^{-bu} - e^{-u(a+c)}) du \right] \\ \left[\sum_{i=1}^{\infty} e^{-\mu x} \frac{(\mu x)^{i-1}}{(i-1)!} q^{i-1} \mu pr_i(y) \right] \end{Bmatrix}$$
$$\left[\sum_{k=0}^{\infty} (F_k(x) - F_{k+1}(x)) g_k(w) \right] \qquad (8)$$

There are two terms inside the flower bracket. The first term presents the pdf part that machine fails when the manpower system is in collapsed state, the repairs and recruitments are taken up. The second term of the flower bracket is the part of the pdf that the man power collapses when the machine is already in failed state. Since hiring is permitted the production continues till the manpower- machine system fails. The last square bracket indicates that k products are produced and sold one by one. Here $r_i(y)$, $(g_k(w))$ indicates the i- fold, (k fold) convolution of r(y), (g(w)) respectively. $F_k(w)$ indicates the k fold Stieltjes convolution of Cdf F(x) with itself.

We now find the quadruple Laplace transform of the pdf as follows.

$$f^*(\xi,\eta,\varepsilon,\delta) = \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty e^{-\xi x - \eta y - \varepsilon z - \delta w} f(x,y,z,w) dxdydzdw \qquad (9)$$

substituting (8) in (9),

$$f^*(\xi,\eta,\varepsilon,\delta)=$$

$$\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty e^{-\xi x-\eta y-\varepsilon z-\delta w}\left\{\begin{array}{l}\left[ae^{-x(a+c)}r_1(z)+\dfrac{cb}{c+a-b}(e^{-bx}-e^{-(c+a)x})r_2(z)\right]\\ \left[\displaystyle\int_0^x\sum_{i=1}^\infty \mu e^{-\mu u}\frac{(\mu u)^{i-1}}{(i-1)!}q^{i-1}pr_i(y)du\right]+\\ \left[r_1(z)\displaystyle\int_0^x ae^{-u(a+c)}du+r_2(z)\frac{cb}{c+a-b}\int_0^x(e^{-bu}-e^{-u(a+c)})du\right]\\ \left[\displaystyle\sum_{i=1}^\infty e^{-\mu x}\frac{(\mu x)^{i-1}}{(i-1)!}q^{i-1}\mu pr_i(y)\right]\end{array}\right\}$$

$$\left[\sum_{k=0}^\infty (F_k(x)-F_{k+1}(x))g_k(w)\right]dxdydzdw$$

Integrating with respect to z and w,

$$f^*(\xi,\eta,\varepsilon,\delta)=$$

$$\int_0^\infty\int_0^\infty e^{-\xi x-\eta y}\left\{\begin{array}{l}\left[ae^{-x(a+c)}r_1^*(\varepsilon)+\dfrac{cb}{c+a-b}(e^{-bx}-e^{-(c+a)x})r_2^*(\varepsilon)\right]\\ \left[\displaystyle\int_0^x \mu e^{-\mu u}\sum_{i=1}^\infty\frac{(\mu u)^{i-1}}{(i-1)!}q^{i-1}pr_i(y)du\right]\\ \left[\displaystyle\sum_{k=0}^\infty (F_k(x)-F_{k+1}(x))g^{*k}(\delta)\right]+\\ \left[\begin{array}{l}\left(\dfrac{a}{a+c}\right)(1-e^{-(a+c)x})r_1^*(\varepsilon)\\ +r_2^*(\varepsilon)\left(\dfrac{cb}{a+c-b}\right)\left[\left(\dfrac{1-e^{-bx}}{b}\right)+\left(\dfrac{e^{-(a+c)x}-1}{a+c}\right)\right]\end{array}\right]\\ \left[\displaystyle\sum_{i=1}^\infty\frac{(\mu x)^{i-1}}{(i-1)!}q^{i-1}pr_i(y)\mu e^{-\mu x}\right]\\ \left[\displaystyle\sum_{k=0}^\infty (F_k(x)-F_{k+1}(x))g^{*k}(\delta)\right]\end{array}\right\}dxdy$$

Integrating with respect to y and u, summing over i,

$$
\begin{aligned}
&f^*(\xi,\eta,\varepsilon,\delta)\\
&=\int_0^\infty e^{-\xi x}\mu pr^*(\eta)\frac{(1-e^{-\mu x(1-qr^*(\eta))})}{\mu(1-qr^*(\eta))}\\
&\left[ae^{-x(a+c)}r_1^*(\varepsilon)+\left(\frac{cb}{c+a-b}\right)(e^{-bx}-e^{-(c+a)x})r_2^*(\varepsilon)\right]\\
&\left[\sum_{n=1}^{\infty}(F_k(x)-F_{k+1}(x))g^{*k}(\delta)\right]dx\\
&+\int_0^\infty e^{-\xi x}\mu pr^*(\eta)e^{-\mu x(1-qr^*(\eta))}\left[\begin{array}{l} r_1^*(\varepsilon)\dfrac{a}{a+c}(1-e^{-x(a+c)})\\ +r_2^*(\varepsilon)\left(\dfrac{cb}{c+a-b}\right)\left(\dfrac{1-e^{-bx}}{b}-\dfrac{1-e^{-(c+a)x}}{c+a}\right)\end{array}\right]\\
&\left[\sum_{n=1}^{\infty}(F_k(x)-F_{k+1}(x))g^{*k}(\delta)\right]dx \qquad (10)
\end{aligned}
$$

This simplifies as follows

$$
\begin{aligned}
&f^*(\xi,\eta,\varepsilon,\delta)=\left[\frac{1}{\chi_4}\frac{1-f^*(\chi_4)}{(1-g^*(\delta)f^*(\chi_4))}-\frac{1}{\chi_1}\frac{1-f^*(\chi_1)}{(1-g^*(\delta)f^*(\chi_1))}\right]\\
&\frac{pr^*(\eta)}{(1-qv^*(\eta))}[ar_1^*(\varepsilon)-\frac{cb}{c+a-b}r_2^*(\varepsilon)]+\\
&\left[\frac{1}{\chi_5}\frac{1-f^*(\chi_5)}{(1-g^*(\delta)f^*(\chi_5))}-\frac{1}{\chi_2}\frac{1-f^*(\chi_2)}{(1-g^*(\delta)f^*(\chi_2))}\right]\\
&\frac{pr^*(\eta)}{(1-qv^*(\eta))}\frac{cb}{c+a-b}r_2^*(\varepsilon)+\\
&\frac{1}{\chi_3}\frac{1-f^*(\chi_3)}{(1-g^*(\delta)f^*(\chi_3))}\mu pr^*(\eta)\left[\frac{a}{a+c}r_1^*(\varepsilon)+\frac{c}{a+c}r_2^*(\varepsilon)\right]\\
&+\frac{1}{\chi_1}\frac{1-f^*(\chi_1)}{(1-g^*(\delta)f^*(\chi_1))}\mu pr^*(\eta)\\
&\left[\frac{-a}{a+c}r_1^*(\varepsilon)+\frac{cb}{(c+a-b)}\frac{1}{(c+a)}r_2^*(\varepsilon)\right]\\
&-\frac{1}{\chi_2}\frac{1-f^*(\chi_2)}{(1-g^*(\delta)f^*(\chi_2))}\mu pr^*(\eta)(\frac{c}{c+a-b})r_2^*(\varepsilon) \qquad (11)
\end{aligned}
$$

Here

$$\chi_1 = \xi + \mu(1 - qr^*(\delta)) + a + c$$
$$\chi_2 = \xi + b + \mu(1 - qr^*(\eta))\ and$$
$$\chi_3 = \xi + \mu(1 - qr^*(\eta))$$
$$\chi_4 = \xi + a + c$$
$$\chi_5 = \xi + b \tag{12}$$

Using differentiation

$$E(X) = -\frac{\partial}{\partial \xi} f^*(\xi,\eta,\varepsilon,\delta)\ at \xi = \eta = \varepsilon = \delta = 0$$

$$This\ gives\ E(X) = \frac{\alpha\mu p}{(a+c)(c+a+\mu p)} + \frac{\beta\mu p}{b(b+\mu p)} + \frac{1}{\mu p} \tag{13}$$

Here α and β are as given in (7).

$$Now \quad E(\hat{R}) = -\frac{\partial}{\partial \eta} f^*(\xi,\eta,\varepsilon,\delta)\ at \xi = \eta = \varepsilon = \delta = 0$$
$$= -\frac{\partial}{\partial \eta} f^*(0,\eta,0,0)\ at \eta = 0$$
$$= -\frac{\partial}{\partial \eta} \frac{pr^*(\eta)}{1 - qr^*(\eta)}\ at \eta = 0$$

After simplification

$$E(\hat{R}) = \frac{E(R)}{p} \tag{14}$$

The expected repair time is

$$E(\tilde{R}) = -\frac{\partial}{\partial \varepsilon} f^*(\xi,\eta,\varepsilon,\delta)\ at\ \xi = \eta = \varepsilon = \delta = 0$$

After simplification

$$E(\tilde{R}) = \frac{a}{c+a}\ E(R_1) + \frac{c}{c+a}\ E(R_2) \tag{15}$$

The Laplace transform of the joint pdf of $(X, \hat{S})$ is given by using (11) as follows.

$$f^*(\xi, 0, 0, \delta) = \frac{\alpha(c+a)}{\chi_4} \frac{(1 - f^*(\chi_4))}{1 - g^*(\delta) f^*(\chi_4)} + \frac{\beta b}{\chi_5} \frac{(1 - f^*(\chi_5))}{1 - g^*(\delta) f^*(\chi_5)} - \frac{\alpha(c + a + p\mu)}{\chi_6} \frac{(1 - f^*(\chi_6))}{1 - g^*(\delta) f^*(\chi_6)} - \frac{\beta(b + p\mu)}{\chi_7} \frac{(1 - f^*(\chi_7))}{1 - g^*(\delta) f^*(\chi_7)} + \frac{p\mu}{\chi_8} \frac{(1 - f^*(\chi_8))}{1 - g^*(\delta) f^*(\chi_8)} \quad (16)$$

Here χ_4 and χ_5 are as given in equation (12) and

$$\chi_6 = \xi + a + c + \mu p$$
$$\chi_7 = \xi + b + \mu p$$
$$\chi_8 = \xi + \mu p \quad (17)$$

Now the joint moment

$E(X \hat{S})$ *and* exp*ected value of* $E(\hat{S})$ *are given by*

$$E(X \hat{S}) = \frac{\partial^2}{\partial \xi \partial \delta} f^*(\xi, 0, 0, \delta) \mid \xi = \delta = 0$$

$$E(\hat{S}) = \frac{\partial^2}{\partial \xi \partial \delta} f^*(\xi, 0, 0, \delta) \mid \xi = \delta = 0$$

Further

$$E(\hat{S}) = E(G) \left[\frac{\alpha f^*(c+a)}{(1 - f^*(c+a))} + \frac{\beta f^*(b)}{(1 - f^*(b))} - \frac{\alpha f^*(c + a + \mu p)}{(1 - f^*(c + a + \mu p))} - \frac{\beta f^*(b + \mu p)}{(1 - f^*(b + \mu p))} + \frac{f^*(\mu p)}{(1 - f^*(\mu p))} \right] \quad (18)$$

and $E(X \hat{S})$ *as follows*

$$E(X\hat{S}) = E(G)\left[\begin{array}{l} \frac{\alpha}{c+a}\frac{f^*(c+a)}{(1-f^*(c+a))} - \frac{\alpha f^{*'}(c+a)}{(1-f^*(c+a))^2} \\ +\frac{\beta}{b}\frac{f^*(b)}{(1-f^*(b))} - \frac{\beta f^{*'}(b)}{(1-f^*(b))^2} \\ +\frac{1}{\mu p}\frac{f^*(\mu p)}{(1-f^*(\mu p))} - \frac{f^{*'}(\mu p)}{(1-f^*(\mu p))^2} \\ -\frac{\alpha}{c+a+\mu p}\frac{f^*(c+a+\mu p)}{(1-f^*(c+a+\mu p))} \\ +\frac{\alpha f^{*'}(c+a+\mu p)}{(1-f^*(c+a+\mu p))^2} - \frac{\beta}{b+\mu p}\frac{f^*(b+\mu p)}{(1-f^*(b+\mu p))} \\ +\frac{\beta f^{*'}(b+\mu p)}{(1-f^*(b+\mu p))^2} \end{array}\right] \quad (19)$$

Since $Cov(X,\hat{S}) = E(X\hat{S}) - E(X)E(\hat{S})$ *equations* (13), (18) *and* (19) *may be used for writing the Co-variance.*

3. Model 2

In this section we treat the previous Model 1 with all assumptions (1), (3), (4), (5) and (6) except the assumption (2) given for sales.

3.1 Assumption For Sales

1. When the operation time X is more than a threshold time U, the sales are done all together. It is assigned to an agent whose sale time distribution function is $G_1(z)$ and pdf $g_1(z)$.

2. When the operation time X is less than a threshold time U, the sales are done one by one with Cdf $G_2(z)$ and pdf $g_2(z)$.

3. The threshold U has exponential distribution function with parameter θ.

3.2 Analysis

Using the arguments given for model 1, we find the joint pdf $(X, \hat{R}, \tilde{R}, \hat{S})$ (Operation time, recruitment time, repair time, sales time) as follows.

$$f(x,y,z,w)=\left\{\begin{array}{l}\left[ae^{-x(a+c)}r_1(z)+\dfrac{cb}{c+a-b}(e^{-bx}-e^{-(c+a)x})r_2(z)\right]\\ \left[\int_0^x\sum_{i=1}^{\infty}\mu e^{-\mu x}\dfrac{(\mu x)^{i-1}}{(i-1)!}q^{i-1}pr_i(y)\right]+\\ \left[r_1(z)\int_0^x ae^{-u(a+c)}du+r_2(z)\dfrac{cb}{c+a-b}\int_0^x(e^{-bu}-e^{-u(a+c)})du\right]\\ \left[\sum_{i=1}^{\infty}e^{-\mu x}\dfrac{(\mu x)^{i-1}}{(i-1)!}q^{i-1}\mu pr_i(y)\right]\end{array}\right\}$$
$$\left\{\sum_{k=0}^{\infty}\left[F_k(x)-F_{k+1}(x)\right]\left[(1-e^{-\theta x})g_1(w)+e^{-\theta x}g_{2k}(w)\right]\right\} \tag{20}$$

The same arguments are given for model 1 for all terms in the first flower bracket. The second flower bracket presents the sales time pdf $g_1(w)$ when (X>U), the operation time is greater than the threshold and presents the pdf $g_2(w)$ when the operation time is less than the threshold (X<U). Here, $g_{2k}(w)$ is k-fold convolution of $g_2(w)$ with itself indicating sales are done one by one.

We now find the quadruple Laplace transform of the pdf as follows.

$$f^*(\xi,\eta,\varepsilon,\delta)=\int_0^\infty\int_0^\infty\int_0^\infty\int_0^\infty e^{-\xi x-\eta y-\varepsilon z-\delta w}f(x,y,z,w)dxdydzdw \tag{21}$$

Equation (21) using equation (20) becomes the sum of two single integrals as given in equation (10) with the replacement of the function $g^{*k}(\delta)$ there by the function

$\left[(1-e^{-\theta x})g_1^{\,*}(\delta)+e^{-\theta x}g_2^{\,*k}(\delta)\right]$ in the two integrals.

The quadruple Laplace transform can now be seen as

$$f^{*}(\xi,\eta,\varepsilon,\delta)=\left[g_{1}^{*}(\delta)\left(\frac{1}{\chi_{4}}-\frac{1}{\chi_{4}+\theta}\right)+\frac{1}{\chi_{4}+\theta}\frac{1-f^{*}(\chi_{4}+\theta)}{(1-g_{2}^{*}(\delta)f^{*}(\chi_{4}+\theta))}\right.$$
$$\left.-g_{1}^{*}(\delta)\left(\frac{1}{\chi_{1}}-\frac{1}{\chi_{1}+\theta}\right)-\frac{1}{\chi_{1}+\theta}\frac{1-f^{*}(\chi_{1}+\theta)}{(1-g_{2}^{*}(\delta)f^{*}(\chi_{1}+\theta))}\right]\left(\frac{pv^{*}(\eta)}{1-qv^{*}(\eta)}\right)$$
$$\left[ar_{1}^{*}(\varepsilon)-\frac{cb}{(c+a-b)}r_{2}^{*}(\varepsilon)\right]$$
$$+\left[\begin{array}{l}g_{1}^{*}(\delta)\left(\frac{1}{\chi_{5}}-\frac{1}{\chi_{5}+\theta}\right)+\frac{1}{\chi_{5}+\theta}\frac{1-f^{*}(\chi_{5}+\theta)}{(1-g_{2}^{*}(\delta)f^{*}(\chi_{5}+\theta))}\\-g_{1}^{*}(\delta)\left(\frac{1}{\chi_{2}}-\frac{1}{\chi_{2}+\theta}\right)-\frac{1}{\chi_{2}+\theta}\frac{1-f^{*}(\chi_{2}+\theta)}{(1-g_{2}^{*}(\delta)f^{*}(\chi_{2}+\theta))}\end{array}\right]$$
$$\left(\frac{pv^{*}(\eta)}{1-qv^{*}(\eta)}\right)\left[\frac{cb}{(c+a-b)}r_{2}^{*}(\varepsilon)\right]+$$
$$\left[g_{1}^{*}(\delta)\left(\frac{1}{\chi_{3}}-\frac{1}{\chi_{3}+\theta}\right)+\frac{1}{\chi_{3}+\theta}\frac{1-f^{*}(\chi_{3}+\theta)}{(1-g_{2}^{*}(\delta)f^{*}(\chi_{3}+\theta))}\right]$$
$$\mu pr^{*}(\eta)\left[\left(\frac{a}{a+c}\right)r_{1}^{*}(\varepsilon)+\left(\frac{c}{c+a}\right)r_{2}^{*}(\varepsilon)\right]+$$
$$\left[g_{1}^{*}(\delta)\left(\frac{1}{\chi_{1}}-\frac{1}{\chi_{1}+\theta}\right)+\frac{1}{\chi_{1}+\theta}\frac{1-f^{*}(\chi_{1}+\theta)}{(1-g_{2}^{*}(\delta)f^{*}(\chi_{1}+\theta))}\right]$$
$$\mu pr^{*}(\eta)\left[\left(\frac{-a}{a+c}\right)r_{1}^{*}(\varepsilon)+\left(\frac{cb}{c+a-b}\right)\left(\frac{1}{c+a}\right)r_{2}^{*}(\varepsilon)\right]$$
$$-\left[g_{1}^{*}(\delta)\left(\frac{1}{\chi_{2}}-\frac{1}{\chi_{2}+\theta}\right)+\frac{1}{\chi_{2}+\theta}\frac{1-f^{*}(\chi_{2}+\theta)}{(1-g_{2}^{*}(\delta)f^{*}(\chi_{2}+\theta))}\right]$$
$$\mu pr^{*}(\eta)r_{2}^{*}(\varepsilon)\frac{c}{(c+a-b)}. \tag{22}$$

Here χ_1, χ_2, χ_3, χ_4 and χ_5 are as given in equation (12).

Since there is only change in the sales pattern $E(X), E(\hat{R})$ and $E(\tilde{R})$ *remain the same as that of model* 1 *equations* (13), (14) *and* (15). *Using equation* (22) *the joint Laplace transform of* $(X, \hat{S})$ *is given by*

$$f^*(\xi,0,0,\delta)=$$
$$g_1^*(\delta)\left\{\begin{array}{l}\frac{\alpha(c+a)}{\chi_4}+\frac{\beta b}{\chi_5}-\frac{\alpha(c+a+p\mu)}{\chi_6}-\frac{\beta(b+p\mu)}{\chi_7}+\frac{p\mu}{\chi_8}\\-\frac{\alpha(c+a)}{\chi_4+\theta}-\frac{\beta b}{\chi_5+\theta}+\frac{\alpha(c+a+p\mu)}{\chi_6+\theta}+\frac{\beta(b+p\mu)}{\chi_7+\theta}-\frac{p\mu}{\chi_8+\theta}\end{array}\right\}$$
$$+\frac{\alpha(c+a)}{\chi_4+\theta}\frac{(1-f^*(\chi_4+\theta))}{1-g_2^*(\delta)f^*(\chi_4+\theta)}+\frac{\beta b}{\chi_5+\theta}\frac{(1-f^*(\chi_5+\theta))}{1-g_2^*(\delta)f^*(\chi_5+\theta)}$$
$$-\frac{\alpha(c+a+p\mu)}{\chi_6+\theta}\frac{(1-f^*(\chi_6+\theta))}{1-g_2^*(\delta)f^*(\chi_6+\theta)}$$
$$-\frac{\beta(b+p\mu)}{\chi_7+\theta}\frac{(1-f^*(\chi_7+\theta))}{1-g_2^*(\delta)f^*(\chi_7+\theta)}+\frac{p\mu}{\chi_8+\theta}\frac{(1-f^*(\chi_8+\theta))}{1-g_2^*(\delta)f^*(\chi_8+\theta)} \qquad (23)$$

$E(\hat{S})$ is given by

$$E(\hat{S})=-\frac{\partial}{\partial\delta}f^*(\xi,0,0,\delta)\,|\,\xi=\delta=0$$

Further,

$$E(\hat{S})=$$
$$E(G_1)\left[1+\frac{\alpha\theta\mu p}{(a+c+\theta)(a+c+\theta+\mu p)}+\frac{\beta\theta\mu p}{(b+\theta)(b+\theta+\mu p)}-\frac{p\mu}{\theta+p\mu}\right]+$$
$$E(G_2)\left[\begin{array}{l}\frac{\alpha(c+a)}{(a+c+\theta)}\frac{f^*(a+c+\theta)}{(1-f^*(a+c+\theta))}+\frac{\beta b}{(b+\theta)}\frac{f^*(b+\theta)}{(1-f^*(b+\theta))}+\\\frac{p\mu}{(\theta+p\mu)}\frac{f^*(\mu p+\theta)}{(1-f^*(\mu p+\theta))}\\-\frac{\alpha(c+a+\mu p)}{(a+c+\theta+\mu p)}\frac{f^*(a+c+\theta+\mu p)}{(1-f^*(a+c+\theta+\mu p))}\\+\frac{\beta(b+\mu p)}{(b+\theta+\mu p)}\frac{f^*(b+\theta+\mu p)}{(1-f^*(b+\theta+\mu p))}\end{array}\right] \qquad (24)$$

The product moment X and $\hat{S}$ is given by

$$E(X\hat{S}) = -\frac{\partial^2}{\partial\xi\partial\delta} f^*(\xi,0,0,\delta) \mid \xi=\delta=0$$

$$E(X\hat{S}) =$$

$$E(G_1)\begin{bmatrix} \frac{\alpha}{(c+a)}+\frac{\beta}{b}-\frac{\alpha}{(c+a+\mu p)}-\frac{\beta}{(b+\mu p)} \\ +\frac{1}{\mu p}-\frac{\alpha(c+a)}{(a+c+\theta)^2}-\frac{\beta b}{(b+\theta)^2} \\ +\frac{\alpha(c+a+\mu p)}{(a+c+\theta+\mu p)^2}+\frac{\beta(b+\mu p)}{(b+\theta+\mu p)^2}-\frac{p\mu}{(\theta+p\mu)^2} \end{bmatrix} +$$

$$E(G_2)\begin{bmatrix} \frac{\alpha(c+a)}{(a+c+\theta)^2}\frac{f^*(a+c+\theta)}{(1-f^*(a+c+\theta))} \\ -\frac{\alpha(c+a)}{(a+c+\theta)}\frac{f^{*'}(a+c+\theta)}{(1-f^*(a+c+\theta))^2}+ \\ +\frac{\beta b}{(b+\theta)^2}\frac{f^*(b+\theta)}{(1-f^*(b+\theta))}-\frac{\beta b}{(b+\theta)}\frac{f^{*'}(b+\theta)}{(1-f^*(b+\theta))^2} \\ -\frac{\alpha(c+a+\mu p)}{(a+c+\theta+\mu p)^2}\frac{f^*(a+c+\theta+\mu p)}{(1-f^*(a+c+\theta+\mu p))} \\ +\frac{\alpha(c+a+\mu p)}{(a+c+\theta+\mu p)}\frac{f^{*'}(a+c+\theta+\mu p)}{(1-f^*(a+c+\theta+\mu p))^2} \\ -\frac{\beta(b+\mu p)}{(b+\theta+\mu p)^2}\frac{f^*(b+\theta+\mu p)}{(1-f^*(b+\theta+\mu p))} \\ +\frac{\beta(b+\mu p)}{(b+\theta+\mu p)}\frac{f^{*'}(b+\theta+\mu p)}{(1-f^*(b+\theta+\mu p))^2} \\ +\frac{p\mu}{(\theta+p\mu)^2}\frac{f^*(\mu p+\theta)}{(1-f^*(\mu p+\theta))}-\frac{p\mu}{(\theta+p\mu)}\frac{f^{*'}(\mu p+\theta)}{(1-f^*(\mu p+\theta))^2} \end{bmatrix} \quad (25)$$

$$Cov(X,\hat{S}) = E(X\hat{S}) - E(X)E(\hat{S})$$

may be given using (13), (24) and (25).

4. Numerical Examples

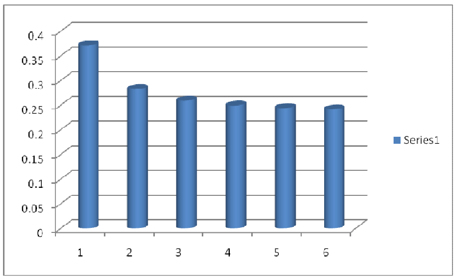
The usefulness of the results obtained is presented by numerical examples. The two Models-1 and 2 are considered together. Since there is only change in sales pattern E(X), $E(\hat{R})$ and **E(** $\tilde{R}$ **)** are same for Models 1and 2.

4.1 Numerical values for Model 1

Let a=6, b=3, c=4, p=0.4, q=0.6, E(R)=20, E(R_1) =10, E(R_2)=5, E(G)=10, μ=10,20,30,40,50,60 λ=2,4,6,8,10,12. Here 'f' is assumed as an exponential density function with parameter 'λ'.

μ	E(X)
10	0.371086
20	0.282572
30	0.259086
40	0.249269
50	0.244197
60	0.241225

Table(i): E(X)

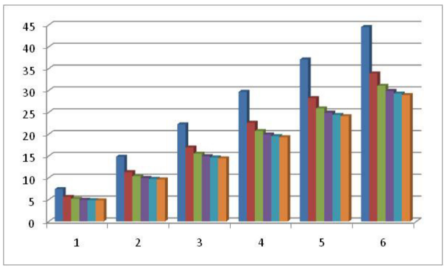


Graph (i)E(X)

From the table and graph it is observed that, when μ increases the expected operation time E(X) decreases.

μ/λ	2	4	6	8	10	12
10	7.421714	14.84343	22.26514	29.68686	37.10857	44.53029
20	5.651434	11.30287	16.9543	22.60574	28.25719	33.90861
30	5.181721	10.36344	15.54516	20.72688	25.90861	31.09033
40	4.985385	9.970769	14.95615	19.94154	24.92692	29.91231
50	4.883942	9.767884	14.65183	19.53577	24.41971	29.30365
60	4.824497	9.648993	14.47349	19.29799	24.12248	28.94698

Table (ii): $E(\hat{S})$

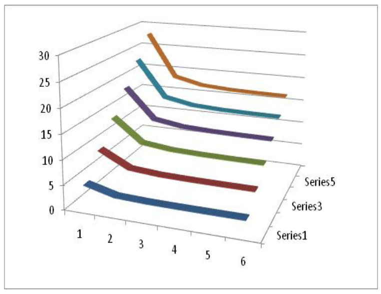


Graph (ii): $E(\hat{S})$

From the table and graph it is observed that, when λ increases the expected sales time $E(\hat{S})$ increases and when μ increases the expected sales time $E(\hat{S})$ decreases.

μ/λ	2	4	6	8	10	12
10	4.657175	3.094299	2.851882	2.778686	2.748855	2.734372
20	9.314351	6.188598	5.703765	5.557371	5.497711	5.468744
30	13.97153	9.282897	8.555647	8.336057	8.246566	8.203116
40	18.6287	12.3772	11.40753	11.11474	10.99542	10.93749
50	23.28588	15.4715	14.25941	13.89343	13.74428	13.67186
60	27.94305	18.56579	17.11129	16.67211	16.49313	16.40623

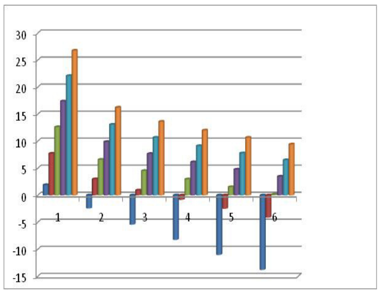
Table(iii): $E(X, \hat{S})$



Graph (iii): $E(X, \hat{S})$

From the table and graph it is observed that, when λ increases the expected product moment of X and $\hat{S}$ $E(X,\hat{S})$ decreases and when μ increases $E(X,\hat{S})$ increases.

μ/λ	2	4	6	8	10	12
10	1.903083	-2.41389	-5.41039	-8.23768	-11.0216	-13.7902
20	7.717415	2.994727	0.912958	-0.83037	-2.48697	-4.11287
30	12.62901	6.597874	4.528112	2.96601	1.534007	0.148046
40	17.386	9.89179	7.679421	6.143931	4.781906	3.48127
50	22.09323	13.08621	10.68148	9.12285	7.781054	6.515993
60	26.77926	16.23822	13.61993	12.01696	10.67419	9.423501



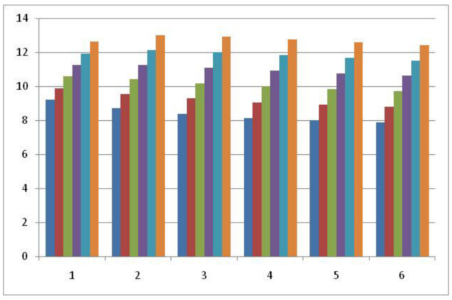
Table(iv): $Cov(X,\hat{S})$ **Graph(iv):** $Cov(X,\hat{S})$

From the table and graph it is observed that when λ increases the Covariance decreases and μ increases, the $Cov(X,\hat{S})$ increases.

4.2 Numerical values for Model 2

Let p = 0.4, a= 6, b=3, c=4, $E(G_1)$ =10, $E(G_2)$=20, μ=10,20,30,40,50,60, λ=2,4,6,8,10,12 and θ=10. Here 'f' is an exponential density function with parameter 'λ'

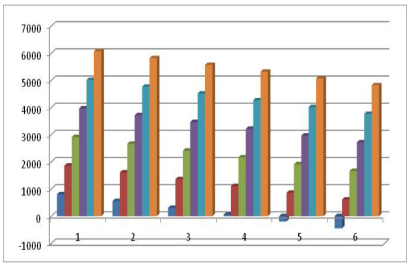
λ/μ	10	20	30	40	50	60
2	9.212839	8.698561	8.3659	8.142113	7.985339	7.871569
4	9.891836	9.555449	9.273663	9.061397	8.902841	8.782898
6	10.57083	10.41234	10.18143	9.980681	9.820342	9.694226
8	11.24983	11.26922	11.08919	10.89997	10.73784	10.60555
10	11.92883	12.12611	11.99695	11.81925	11.65534	11.51688
12	12.60783	12.983	12.90472	12.73853	12.57285	12.42821



Table(v): $E(\hat{S})$ **Graph(v):** $E(\hat{S})$

From the table and graph it is observed that, when λ increases the expected sales time $E(\hat{S})$ increases and when μ increases the expected sales time $E(\hat{S})$ decreases.

λ/μ	10	20	30	40	50	60
2	803.6678	553.9906	304.148	54.24402	-195.69	-445.642
4	1856.532	1606.858	1357.003	1107.088	857.1452	607.1869
6	2909.397	2659.725	2409.859	2159.932	1909.981	1660.016
8	3962.261	3712.592	3462.714	3212.777	2962.816	2712.844
10	5015.125	4765.459	4515.569	4265.621	4015.651	3765.673
12	6067.99	5818.326	5568.425	5318.465	5068.487	4818.501



Table(vi): $E(X, \hat{S})$

Graph (vi): $E(X, \hat{S})$

From the table and graph it is observed that, when λ increases $E(X, \hat{S})$ increases and when μ increases $E(X, \hat{S})$ decreases.

λ/μ	10	20	30	40	50	60
2	800.249	550.7627	301.0435	51.2226	-198.653	-448.563
4	1853.737	1604.158	1354.383	1104.528	854.6295	604.7051
6	2906.658	2657.027	2407.221	2157.346	1907.436	1657.504
8	3959.457	3709.783	3459.95	3210.059	2960.139	2710.201
10	5012.212	4762.498	4512.64	4262.734	4012.805	3762.86
12	6064.949	5815.194	5565.312	5315.392	5065.454	4815.503

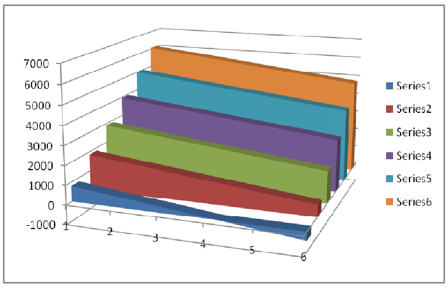


Table (vii): $Cov(X, \hat{S})$

Graph(vii): $Cov(X, \hat{S})$

From the table it is observed that when λ increases, the $Cov(X, \hat{S})$ increases and when μ increases the $Cov(X, \hat{S})$ decreases.

Conclusion

Manpower system exposed to a departure process has been treated assuming the machine producing products for sale has SCBZ failure pattern. Repair times of the machine, recruitment times of employees and sales time of products have been assumed to have general distributions. In model 1, the sales are done one by one. It is observed that when the departure rate of employees increases the expected operation time and the expected sales time decreases. The product moment of operation time and sales time increases when the departure rate of employees increases. The covariance of operation time and sales time increases when the inter departure rate of employees increases. In model 2, when the operation time is more than a threshold, the sales are done altogether and when it is less than the threshold, the sales are done one by one. It is observed that when the inter departure rate of employees increases the expected sales time, the product moment of operation time and sales time, covariance of operation time and sales time decrease. Noting the SCBZ failure pattern in phase 2 Coxian, it may be useful if further research is taken up for Coxian with higher phases.

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