Neutrino Oscillation and CPT Violation: A Four Flavour Matter Effects

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Abstract

In a matter for the four flavour neutrino oscillation (3+1) scheme, the CPT violation is taken into consideration. **Assuming** that the mismatch between neutrino and anti-neutrino mass square discrepancies causes the CPT to be violated in matter. We suppose that the presence of matter causes the four flavour neutrino and anti-neutrino mass differences to square. For four flavour neutrino oscillation, we take into account the pertinent matter density profile. In this study, we calculate the upper bound constraint of the CPT violation in matter for a four flavour scheme. $|\Delta_{21}^m - \overline{\Delta_{21}^m}| = (1.21 - 3.44) \times 10^{-3})eV^2, |\Delta_{31}^m - \overline{\Delta_{31}^m}| = (1.68 - 6.41) \times 10^{-4})eV^2$ and $|\Delta_{41}^m - \overline{\Delta_{41}^m}| = (1.06 - 4.60) \times 10^{-5})eV^2$.

In this paper, we examine the CPT violation in a framework with four flavours in matter.

Keywords: CPT violation, neutrino oscillation.

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1 Introduction

Solar [1], atmospheric [2], reactor [3], and long-baseline neutrino experiments [4, 5] have all established the existence of neutrino oscillation. Results from LSND and MiniBoone indicate the existence of sterile neutrinos [7, 8]. The solar neutrino mass square difference is measured by the LSND experiment to be $0.2 < \Delta_{21} < 100eV^2$ [6] and the MiniBooNe experiment to be $0.01 < \Delta_{21} < 1.0eV^2$. These two investigations provide up new avenues for sterile neutrinos. By taking into account how neutrinos interact with matter, one can alter the likelihood of neutrinos oscillating in a vacuum [8]. One of the fundamental symmetries in particle physics is the CPT. The equality of the survival probabilities of neutrinos and anti-neutrinos in vacuum is implied by neutrino oscillation according to CPT conservation [9, 10]. Neutrino oscillation as a method of looking for CPT violations was initially put forth in Ref. [9]. Constraints on the solar and KamLAND data.

There is a signal for a CPT violation in the neutrino sector when $|\Delta_{21} - \overline{\Delta_{21}}| \leq 1.1 \times 10^{-4} eV^2$[11], which is a non-zero difference between Δ_{21} and $\overline{\Delta_{21}}|$. Prior to now, the majority of research on CPT violation [12–19]. For a four-flavor matter paradigm, we provide analytical expressions for three distinct neutrino mass square differences. In this essay, we would look at a potential CPT violation in a four-flavor framework. The following is a summary of the article. We talk about four flavour neutrino oscillation in vacuum with sterile neutrino in Section 2 of this article. We briefly touch on the CPT Violation four flavour neutrino oscillation in Section 3. Section 4 contains numerical results and Section 5 for the conclusion.

2 Four Flavor Neutrino Oscillation in Vacuum

In this part, we assume the sterile neutrino mix contains three distinct neutrinos and consider with four flavour neutrino oscillation. The PMNS matrix $U_{4\times 4}$ is provided by [20], and by adding one sterile neutrino [21],

$$U = R_{34}(\theta_{34}, \delta_{34}) R_{24}(\theta_{34}) R_{14}(\theta_{14}, \delta_{14}) R_{23}(\theta_{23}) R_{13}(\theta_{13}, \delta_{13}) R_{12}(\theta_{12}), \tag{1}$$

The rotations in ij space represented by the matrices R_{ij}

$$R_{ij}(\theta_{ij}, \delta) = \begin{pmatrix} c_{ij} & s_{ij}e^{-i\delta} \\ -s_{ij}e^{i\delta} & c_{ij} \end{pmatrix},$$

where $s_{ij} = sin\theta_{ij}, c_{ij} = cos\theta_{ij}$.

In four flavor there are three Dirac CP-violating phase δ_{ij}. The explicit form of U is

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{pmatrix}, \qquad (2)$$

where

$$U_{e1} = (c_{14}c_{13}c_{12}),$$

$$U_{e2} = (c_{14}c_{13}s_{12}),$$

$$U_{e3} = (c_{14}s_{13})e^{-i\delta_{13}},$$

$$U_{e4} = s_{14}e^{i\delta_{14}}e^{if_1},$$

$$U_{\mu 1} = (-s_{24}c_{14}c_{13}e^{i\delta_{14}} - c_{24}s_{23}s_{13}c_{12}e^{i\delta_{13}})c_{12} - c_{24}c_{23}s_{12}e^{-i\delta},$$

$$U_{\mu 2} = (c_{24}c_{23}c_{12} - s_{12}(s_{24}s_{14}c_{13}e^{i\delta_{14}} + c_{24}s_{23}s_{13}e^{-i\delta_{13}}),$$

$$U_{\mu 3} = c_{24}s_{23}c_{13} - s_{24}s_{14}c_{13}e^{-i\delta(\delta_{14}-\delta_{13})},$$

$$U_{\mu 4} = (s_{14})e^{-i\delta_{14}},$$

$$U_{\tau 1} = c_{12}[-s_{34}c_{24}s_{14}c_{13}e^{-i(\delta_{34}-\delta_{14})} - s_{13}e^{i\delta_{13}}(c_{34}c_{23} - s_{34}s_{24}s_{23}e^{-i\delta_{34}})] \\ + s_{12}(s_{34}s_{24}c_{23}e^{-i\delta_{34}} + s_{12}(s_{34}s_{24}c_{23}e^{-i\delta_{34}} + c_{34}s_{23}),$$

$$U_{\tau 2} = -c_{12}(s_{34}s_{24}c_{23}e^{-i\delta_{34}} + c_{34}s_{23}) - s_{12}[s_{34}c_{24}s_{14}s_{13}e^{-i(\delta_{34}-\delta_{14})} \\ + s_{13}e^{i\delta_{13}}(c_{34}c_{23} - s_{34}s_{24}s_{23}e^{-i\delta_{34}})],$$

$$U_{\tau 3} = -c_{13}(c_{34}c_{23} - s_{34}s_{24}s_{23}e^{-i\delta_{34}}) - s_{34}c_{24}s_{14}s_{13}e^{-i(\delta_{34}-\delta_{13})},$$

$$U_{\tau 4} = (s_{34}c_{24}c_{14})e^{i\delta_{34}},$$

$$U_{s1} = c_{12}[-c_{34}c_{24}s_{14}c_{13}e^{i\delta_{14}} + s_{13}e^{i\delta_{13}}(s_{34}c_{23}e^{i\delta_{34}} + c_{34}s_{24}s_{23})] \\ - s_{12}(-c_{34}s_{24}c_{23} + s_{34}s_{23}e^{i\delta_{34}})$$

$$U_{s2} = c_{12}(-c_{34}s_{24}c_{23} + s_{34}s_{23}e^{i\delta_{34}})$$

$$U_{s3} = (-s_{34}c_{23}c_{13} - c_{34}s_{24}s_{23}c_{13} - c_{34}c_{24}s_{14}s_{13}e^{-i\delta})e^{i\gamma}e^{if_4}$$

$$U_{s4} = (c_{34}c_{24}c_{14})e^{if_4},$$

If we suppose that there is one sterile neutrino, then [21] gives the probability of four flavour neutrino oscillation in a vacuum

$$P_{\nu_\alpha \to \nu_\beta} = \sum_i |U_{\mu 1}|^2 |U_{e1}|^2 + 2\sum_{i<j} [Re(U_{ei}U_{\mu j}U^*_{ej}U^*_{\mu i}) cos\bar{\Delta}_{ij} \\ - Im(U_{ei}U_{\mu j}U^*_{ej}U^*_{\mu i}) sin\bar{\Delta}_{ij}], \quad (3)$$

where $\bar{\Delta}_{ij} = \Delta_{ij} L/2E$, baseline length of particular experiment is L.

3 Neutrino Mass Square Difference in Matter for Four Flavor Framework

The Hamiltonian H_{vacuum} in the propagation of neutrinos in vacuum is given by

$$H_{vacuum} = \begin{pmatrix} E_1 & 0 & 0 & 0 \\ 0 & E_2 & 0 & 0 \\ 0 & 0 & E_3 & 0 \\ 0 & 0 & 0 & E_4 \end{pmatrix}, \quad (4)$$

where $E_k (k = 1, 2, 3, 4)$ are the energies of the neutrino mass eigenstates k with mass m_k;

$$E_k = \sqrt{m_k^2 + p^2}, \quad (5)$$

We assume that all mass eigenstates have the same momentum, p. when matter and neutrinos interact weakly (charged and neutral current). The sterile neutrino does not participate in any weak interactions. For mixing of four flavours of neutrinos, the effective Hamiltonian is [12].

$$H_{eff} = \frac{1}{2E} \left[U \begin{pmatrix} m_1^2 & 0 & 0 & 0 \\ 0 & m_2^2 & 0 & 0 \\ 0 & 0 & m_3^2 & 0 \\ 0 & 0 & 0 & m_4^2 \end{pmatrix} U^\dagger + \begin{pmatrix} A & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & A' \end{pmatrix} \right], \quad (6)$$

where U is four flavor mixing matrix and A and A' is matter dependent term is given by

$$A(eV^2) \quad \begin{matrix} 2\sqrt{2} G_F N_e E_\nu & (neutrinos) \\ -2\sqrt{2} G_F N_e E_\nu & (anti-neutrinos) \end{matrix}$$

and

$$A'(eV^2) \quad \begin{array}{ll} -\sqrt{2}G_F N_n E_\nu & (neutrinos) \\ \\ \sqrt{2}G_F N_n E_\nu & (anti-neutrinos) \end{array}$$

where N_e and N_n is the density of electron and neutron. From Eq. (4.0), we have

$$\begin{aligned} &\left[U \begin{pmatrix} m_1^2 & 0 & 0 & 0 \\ 0 & m_2^2 & 0 & 0 \\ 0 & 0 & m_{3^2} & 0 \\ 0 & 0 & 0 & m_4^2 \end{pmatrix} U^\dagger\right] \\ &= U_m = \left[U \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \Delta_{21} & 0 & 0 \\ 0 & 0 & \Delta_{31} & 0 \\ 0 & 0 & 0 & \Delta_{41} \end{pmatrix} U^\dagger\right] \\ &+ \left[\begin{pmatrix} A & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & A' \end{pmatrix}\right], \end{aligned} \tag{7}$$

where Δ_{21},Δ_{31} and Δ_{41} are the mass squared differences of solar, atmospheric, and sterile neutrinos.

The matrix U_m can be diagonalized to determine the effective mass square difference in matter [20]. The roots of Equation (7.0) yield the matter-dependent mass squares $m_{m1}^2, m_{m2}^2, m_{m3}^2$ and m_{m4}^2

$$\lambda_1 = m_{m1}^2 = -\frac{b}{4} - S - \frac{1}{2}\sqrt{-4S^2 - 2p + \frac{q}{S}},$$

$$\lambda_2 = m_{m2}^2 = -\frac{b}{4} - S + \frac{1}{2}\sqrt{-4S^2 - 2p + \frac{q}{S}},$$

$$\lambda_3 = m_{m3}^2 = -\frac{b}{4} + S - \frac{1}{2}\sqrt{-4S^2 - 2p - \frac{q}{S}},$$

$$\lambda_4 = m_{m4}^2 = -\frac{b}{4} + S + \frac{1}{2}\sqrt{-4S^2 - 2p - \frac{q}{S}}$$

We may write the matter dependent mass square difference for the four flavour neutrino oscillation using the matter dependent mass squares $m^2_{m1}, m^2_{m2}, m^2_{m3}$ and m^2_{m4}.

$$\Delta^m_{21} = m^2_{m2} - m^2_{m1} = \sqrt{-4S^2 - 2p + \frac{q}{S}}, \tag{8}$$

$$\Delta^m_{31} = m^2_{m3} - m^2_{m1} = 2S + \frac{1}{2}\left(\sqrt{-4S^2 - 2p + \frac{q}{S}} - \sqrt{-4S^2 - 2p - \frac{q}{S}}\right), \tag{9}$$

$$\Delta^m_{41} = m^2_{m4} - m^2_{m1} = 2S + \frac{1}{2}\left(\sqrt{-4S^2 - 2p + \frac{q}{S}} + \sqrt{-4S^2 - 2p - \frac{q}{S}}\right). \tag{10}$$

We can write matter-dependent anti-neutrino mass square difference for four flavour neutrino oscillation when anti-neutrino potential term A is replaced by -A.

$$\overline{\Delta^m_{21}} = m^2_{m2} - m^2_{m1} = \sqrt{-4S'^2 - 2p' + \frac{q'}{S'}}, \tag{11}$$

$$\overline{\Delta^m_{31}} = m^2_{m3} - m^2_{m1} = 2S' + \frac{1}{2}\left(\sqrt{-4S'^2 - 2p' + \frac{q'}{S'}} - \sqrt{-4S'^2 - 2p' - \frac{q'}{S'}}\right), \tag{12}$$

$$\overline{\Delta^m_{41}} = m^2_{m4} - m^2_{m1} = 2S' + \frac{1}{2}\left(\sqrt{-4S'^2 - 2p' + \frac{q'}{s'}} + \sqrt{-4S'^2 - 2p' - \frac{q'}{S'}}\right). \tag{13}$$

The density and neutrino oscillation parameter affect p, p', q, q' S and S' [18]. The mass squared difference for neutrinos is given by Equations (8) through (10); meanwhile, Equations (11) through (13) give the mass squared difference for anti-neutrinos of matter terms A and $A^{\cdot}$. Then, the following provides the CPT violating survival probability difference in the four flavour framework:

$$\Delta P_{\alpha\beta}^{CPT} = P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu_\alpha} \to \bar{\nu_\beta}), \tag{14}$$

where $\alpha, \beta = e, \mu, \tau, s$

If CPT conserves in a vacuum, one has $\Delta P_{\alpha\beta}^{CPT} = 0$.

In the event that CPT violated [25, 26]

$$\Delta^m P_{\alpha\beta}^{CPT} \neq 0,$$

$$\Delta_{21}^m(CPT) = \Delta_{21}^m - \overline{\Delta_{21}^m} \neq 0 \tag{15}$$

$$\Delta_{31}^m(CPT) = \Delta_{31}^m - \overline{\Delta_{31}^m} \neq 0 \tag{16}$$

$$\Delta_{41}^m(CPT) = \Delta_{41}^m - \overline{\Delta_{41}^m} \neq 0 \tag{17}$$

4 Numerical Results

The neutrino mass square difference, a CPT violation term for four flavours of neutrino oscillation, is

$$\Delta_{21}^m(CPT) = \sqrt{-4S^2 - 2p + \frac{q}{S}} - \sqrt{-4S'^2 - 2p' + \frac{q'}{S'}}, \tag{18}$$

$$\begin{aligned}\Delta_{31}^m(CPT) = 2(S - S') & \\ & + \frac{1}{2}\left(\sqrt{-4S^2 - 2p + \frac{q}{S}} - \sqrt{-4S'^2 - 2p' + \frac{q'}{S'}}\right) \\ & - \frac{1}{2}\left(\sqrt{-4S^2 - 2p - \frac{q}{S}} - \sqrt{-4S'^2 - 2p' - \frac{q'}{S'}}\right),\end{aligned} \tag{19}$$

$$\begin{aligned}\Delta_{41}^m(CPT) = 2(S - S') & \\ & + \frac{1}{2}\left(\sqrt{-4S^2 - 2p + \frac{q}{S}} - \sqrt{-4S'^2 - 2p' + \frac{q'}{S'}}\right) \\ & + \frac{1}{2}\left(\sqrt{-4S^2 - 2p - \frac{q}{S}} - \sqrt{-4S'^2 - 2p' - \frac{q'}{S'}}\right),\end{aligned} \tag{20}$$

In this computation, we're assuming that the mass order is normal. We presummate matter densities for electron $\rho_e = 3g/cm^3$ and

Table 1 For the matter effect-related neutrino mass square difference and anti-neutrino mass square difference. Having taken $\Delta_{31} = 2.0 \times 10^{-3} eV^2$, $\Delta_{21} = 8.0 \times 10^{-5} eV^2$, $\Delta_{41} = 1.7 eV^2$ and mixing angles $\theta_{13} = 10°, \theta_{23} = 45°, \theta_{12} = 34°$,$\theta_{34} = 18.5°, \theta_{24} = 4°, \theta_{14} = 3.6°$

Δ^m_{ij}	Energy	$\Delta^m_{ij}(CPT) = \lvert\Delta^m_{ij}$-$\overline{\Delta^m_{ij}}\rvert$
Δ^m_{21}	1	3.44×10^{-3}
Δ^m_{31}	1	1.52×10^{-4}
Δ^m_{41}	1	4.47×10^{-5}
Δ^m_{21}	2	7.76×10^{-3}
Δ^m_{31}	2	3.69×10^{-4}
Δ^m_{41}	2	2.66×10^{-5}
Δ^m_{21}	3	1.21×10^{-3}
Δ^m_{31}	3	5.94×10^{-4}
Δ^m_{41}	3	3.73×10^{-6}

$\rho_n = 3g/cm^3$ **for calculation of neutrino mass square differences in matter.** $\Delta^m_{21}, \Delta^m_{31}$ and Δ^m_{41} are modified mass square differences in matter that depend on the matter density, the six neutrino mixing angles, and the Dirac phases δ_{34}, δ_{13}, and δ_{14}. We select the following mixing angles: $\theta_{12} = 34°$, $\theta_{23} = 45°$, $\theta_{13} = 10°$ abandon majorana stages, etc. We examine the following value for sterile neutrino mixing angles in this paper [22], where $\theta_{14} = 3.6°$, $\theta_{24} = 4°$, $\theta_{34} = 18.5°$ and $\Delta_{31} = 0.002eV^2$ [23], $\Delta_{41} = 1.7eV^2$ [22] and $\Delta_{21} = 0.00008eV^2$ [2] have been taken. Table (1.0) lists the maximum differences in neutrino and anti-neutrino mass square differences in matter for four flavour frames in the region of 1 to 3 GeV for various values of Dirac phases $\delta_{34} = 0° < \delta_{34} < 180°$, $\delta_{13} = 0° < \delta_{13} < 180°$, $\delta_{14} = 0° < \delta_{14} < 180°$.

5 Conclusions

We have taken into account potential CPT violations in four different frame works before coming to this result. We started by calculating the squared mass differences of all four flavours of neutrinos in matter. We found that the four flavour frame work neutrino and antineutrino mass square differences are not the same. The maximum change of solar mass square difference and anti-neutrino solar mass square difference in matter is predicted by our numerical study. $\Delta^m_{21} - \overline{\Delta^m_{21}}$=$(1.21 - 3.44) \times 10^{-3})eV^2$ for $\delta_{34} = \delta_{13} = \delta_{14} = 0, 45, 90, 135, 180$ degrees in region 1 to 3 GeV energy region.

Due to matter effects, maximum change of atmospheric mass square difference in matter $\Delta_{31}^{m} - \overline{\Delta_{31}^{m}} = (1.68 - 6.41) \times 10^{-4})eV^2$ for $\delta_{34} = \delta_{13} = \delta_{14} = 0, 45, 90, 135, 180$ degrees in energy region 1 to 3 GeV and maximum change of sterile neutrino mass square difference in matter $\Delta_{41}^{m} - \overline{\Delta_{41}^{m}} = (1.06 - 4.60) \times 10^{-5})eV^2$ for $\delta_{34} = \delta_{13} = \delta_{14} = 0, 45, 90, 135, 180$ degrees in energy region 1 to 3 GeV . We note that $\Delta_{21}^{m}(CPT) = \Delta_{21}^{m} - \overline{\Delta_{21}^{m}}$ contributes significantly to CPT violation in four flavour neutrino oscilation in matter.

Data Availability Statement

No Data associated in the manuscript.

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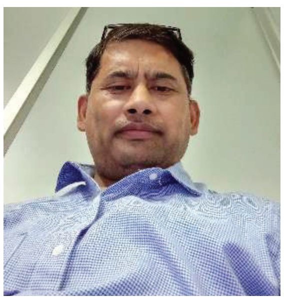
Biographies



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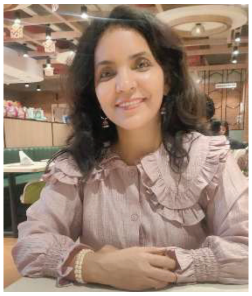

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